BBM Equation

```
> with(Exterior):
```

Exterior calculus package, version 1.12 (30 Oct 2009).
 (1)

```
> deq:=Diff(u,x,x,t)=Diff(u,t)-u*Diff(u,x);
```

$$deq := \frac{\partial^3}{\partial t\, \partial x^2} u = \frac{\partial}{\partial t} u - u \left(\frac{\partial}{\partial x} u \right) \tag{2}$$

```
> find_symmetry(deq);
```

$$\begin{bmatrix} \partial_x \\ \partial_t \\ -t\, \partial_t + u\, \partial_u \end{bmatrix} \tag{3}$$

```
> clear(): #This restarts Exterior
```

1-D Nonlinear Wave Equation

```
> depend([u],f):
> deq:=Diff(u,t$2)=diff(f,u)*Diff(u,x)^2+f*Diff(u,x$2); # Note it is diff(f,u) not Diff(f,u)
```

$$deq := \frac{\partial^2}{\partial t^2} u = \left(\frac{\partial}{\partial u} f \right) \left(\frac{\partial}{\partial x} u \right)^2 + f \left(\frac{\partial^2}{\partial x^2} u \right) \tag{4}$$

```
> symmetry,eq:=find_symmetry(deq,casesplit):
> caseplot(eq,pivots);
```

========= *Pivots Legend* =========

$$p1 = f$$

$$p2 = \frac{\partial^2}{\partial u^2} f$$

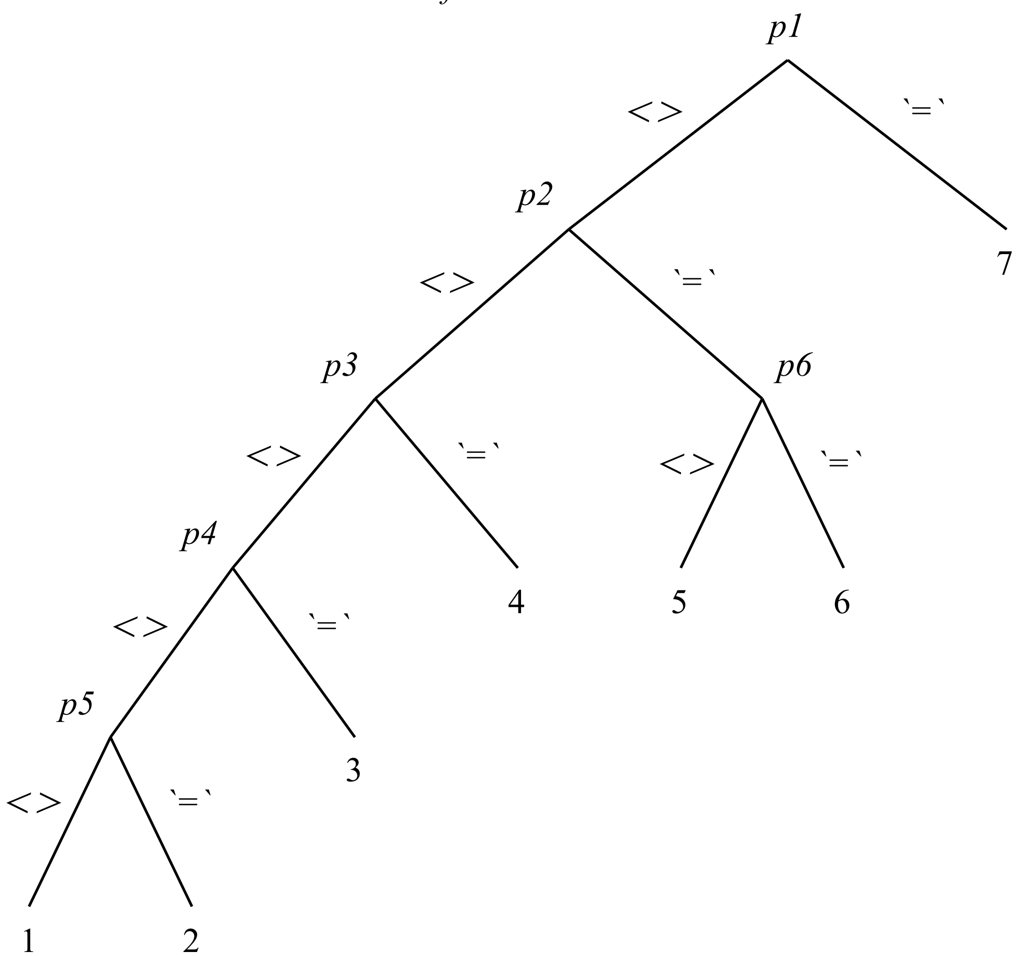
$$p3 = -4\, f \left(\frac{\partial^2}{\partial u^2} f \right) + 5 \left(\frac{\partial}{\partial u} f \right)^2$$

$$p4 = -4\, f \left(\frac{\partial^2}{\partial u^2} f \right) + 7 \left(\frac{\partial}{\partial u} f \right)^2$$

$$p5 = -2\, f \left(\frac{\partial^2}{\partial u^2} f \right)^2 + \left(\frac{\partial}{\partial u} f \right)^2 \left(\frac{\partial^2}{\partial u^2} f \right) + f \left(\frac{\partial}{\partial u} f \right) \left(\frac{\partial^3}{\partial u^3} f \right)$$

$$p6 = \frac{\partial}{\partial u} f$$

Rif Case Tree




```
> group_size(eq);
```

$1 = [\mathit{Finite} = 3, \mathit{Infinite} = [0]], 2 = [\mathit{Finite} = 4, \mathit{Infinite} = [0]], 3 = [\mathit{Finite} = 5, \mathit{Infinite} = [0]], 4 = [\mathit{Finite} = 5, \mathit{Infinite} = [0]], 5 = [\mathit{Finite} = 4, \mathit{Infinite} = [0]], 6 = [\mathit{Finite} = 1, \mathit{Infinite} = [4]], 7 = [\mathit{Finite} = 0, \mathit{Infinite} = [9]]$ **(5)**

We see that there in a infinite dimensional component to the symmetry group in cases 6 and 7 (these are the cases where f is constant and so the equation is linear).

```
> soln:=rifsolve(eq,Parameters={f}):
> G:=one_parameter(symmetry,soln);
```

$$G := table\left(\left[1 = \left[\left[\begin{array}{c} \partial_x \\ \partial_t \\ t\,\partial_t + x\,\partial_x \end{array}\right]\right], 2 = \left[\left[\begin{array}{c} \partial_x \\ \partial_t \\ t\,\partial_t + x\,\partial_x \\ \frac{1}{2}\,t\,\_A1\,\partial_t + (-u + \_A2)\,\partial_u \end{array}\right], [f = (-u\right.\right.$$ **(6)**

$$+\_A2)^{-\_A1}\ \_A3\Big]\Bigg], 3 = \left[\left[\begin{array}{c} \partial_x \\ \partial_t \\ t\,\partial_t + x\,\partial_x \\ \frac{2}{3}\,t\,\_A1\,\partial_t + (\_A1\,u + \_A2)\,\partial_u \\ -\frac{1}{3}\,\_A1\,x^2\,\partial_x + x\,(\_A1\,u + \_A2)\,\partial_u \end{array}\right], \Big[f =\right.$$

$$\left.-\frac{4}{3}\,\frac{2^{2/3}\left(-\frac{1}{3\,(\_A1\,u + \_A2)}\right)^{1/3}}{\_A1\,u + \_A2}\right]\Bigg], 5 = \left[\left[\begin{array}{c} \partial_x \\ \partial_t \\ t\,\partial_t + x\,\partial_x \\ -\frac{1}{2}\,t\,\_A1\,\partial_t + (\_A1\,u + \_A2)\,\partial_u \end{array}\right], [f\right.$$

$$= \_A1\,u + \_A2]\Bigg], 4 = \left[\left[\begin{array}{c} \partial_x \\ \partial_t \\ t\,\partial_t + x\,\partial_x \\ 2\,t\,\_A1\,\partial_t + (\_A1\,u + \_A2)\,\partial_u \\ \_A1\,t^2\,\partial_t + t\,(\_A1\,u + \_A2)\,\partial_u \end{array}\right], \left[f = \frac{256}{(\_A1\,u + \_A2)^4}\right]\right], 7$$

$$= \left[[\ \ ], \left(\frac{1}{2}\,u\,\_F5(x)\,t + u\,\_F11(x) + t^2\,\_F3(x) + \_F12(x)\,t + \_F13(x)\right)\partial_t\right.$$

$$+ \_F8(x)\,\partial_x + \left(t\,\_F3(x)\,u + t\,\_F4(x) + \frac{1}{2}\,\_F5(x)\,u^2 + \_F6(x)\,u + \_F7(x)\right)\partial_u, [f$$

$$= 0]\Big], 6 = \left[\left[\begin{array}{c} u\,\partial_u \\ \partial_x \end{array}\right], \left(\_F5\left(x + \sqrt{\_A1}\,t\right) + \_F6\left(x - \sqrt{\_A1}\,t\right)\right)\partial_t + \left(\sqrt{\_A1}\,\_F5\left(x\right.\right.\right.$$

$$\left.+\sqrt{\_A1}\,t\right) - \sqrt{\_A1}\,\_F6\left(x - \sqrt{\_A1}\,t\right)\Big)\,\partial_x + \Big(\_F3\left(x + \sqrt{\_A1}\,t\right) + \_F4\Big(x$$

$$\left.-\sqrt{\_A1}\,t\right)\Big)\,\partial_u, [f = \_A1]\Big]\Big]\Big)$$

Note the infinite dimensional pieces for cases 6 and 7.