Kamke Example 11

```
> with(Exterior):
```

Exterior calculus package, version 1.12 (30 Oct 2009).

 (1)

```
> deq:=Diff(y,x,x)=-a*x^r*y^n;
```

$$deq := \frac{\partial^2}{\partial x^2} y = -a\, x^r\, y^n \tag{2}$$

```
> symmetry,eq:=find_symmetry(deq,Constraints={a<>0,n<>0,n<>1},
  casesplit):
```

Constraints remove the linear cases.

```
> caseplot(eq,pivots);
```

========= *Pivots Legend* =========

$p1 = -2 + n$

$p2 = n + 3$

$p3 = r\,(r + 2)\,(r + n + 3)$

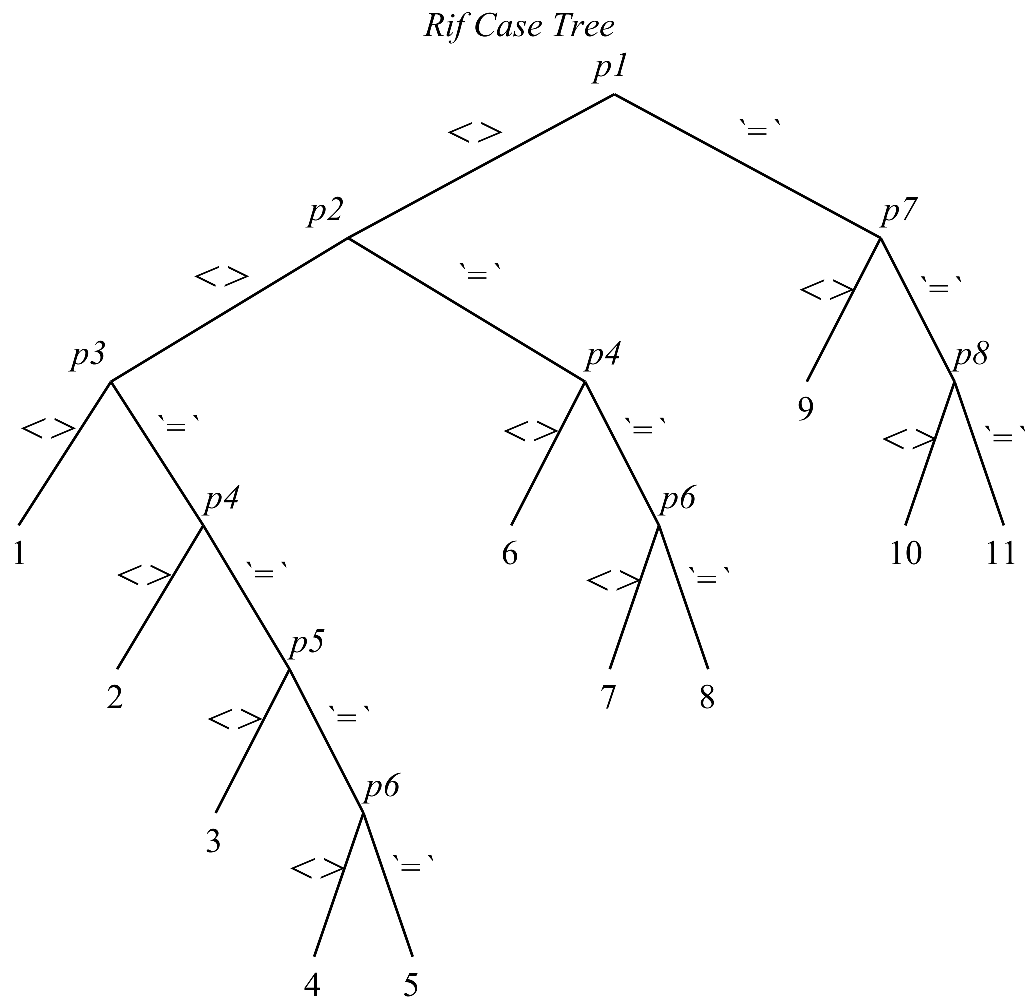
$p4 = r\,(r + 2)$

$p5 = r\,(1 + n)$

$p6 = r$

$p7 = r\,(7\,r + 15)\,(7\,r + 20)\,(r + 5)\,(r + 2)$

$p8 = r\,(7\,r + 15)\,(7\,r + 20)\,(r + 5)$




```
> group_size(eq);
```

$$1 = [Finite = 1, Infinite = [0]], 2 = [Finite = 2, Infinite = [0]], 3 = [Finite = 1, Infinite = [0]], 4 = [Finite = 2, Infinite = [0]], 5 = [Finite = 2, Infinite = [0]], 6 = [Finite = 1, Infinite = [0]], 7 = [Finite = 1, Infinite = [0]], 8 = [Finite = 3, Infinite = [0]], 9 = [Finite = 1, Infinite = [0]], 10 = [Finite = 1, Infinite = [0]], 11 = [Finite = 2, Infinite = [0]] \quad (3)$$

Note that computing the group size does not require solving the determining equations.

```
> soln:=rifsolve(eq):
```

```
> G:=one_parameter(symmetry,soln):
```

```
> for i from 1 to soln[casecount]-1 do i=G[i],Commutator(G[i][1]) od;
```

$$1 = \left[\left[y\,\partial_y - \frac{x\,(n-1)\,\partial_x}{r+2}\right]\right], [\,0\,]$$

$$2 = \left[\left[\begin{array}{c} y\,x\,\partial_y + x^2\,\partial_x \\ \dfrac{y\,(r+2)\,\partial_y}{4+r} + x\,\partial_x \end{array}\right], [n = -r-3]\right], \left[\begin{array}{cc} 0 & -y\,x\,\partial_y - x^2\,\partial_x \\ y\,x\,\partial_y + x^2\,\partial_x & 0 \end{array}\right]$$

$$3=\left[\left[\begin{array}{c} x\,\partial_x \end{array}\right],[r=-2]\right],\left[\begin{array}{c}0\end{array}\right]$$

$$4=\left[\left[\begin{array}{c} y\,x\,\partial_y+x^2\,\partial_x \\ x\,\partial_x \end{array}\right],[n=-1,r=-2]\right],\left[\begin{array}{cc} 0 & -y\,x\,\partial_y-x^2\,\partial_x \\ y\,x\,\partial_y+x^2\,\partial_x & 0 \end{array}\right]$$

$$5=\left[\left[\begin{array}{c} \partial_x \\ y\,\partial_y+\left(-\frac{1}{2}\,x\,n+\frac{1}{2}\,x\right)\partial_x \end{array}\right],[r=0]\right],\left[\begin{array}{cc} 0 & \left(-\frac{1}{2}\,n+\frac{1}{2}\right)\partial_x \\ \left(\frac{1}{2}\,n-\frac{1}{2}\right)\partial_x & 0 \end{array}\right]$$

$$6=\left[\left[\begin{array}{c} y\,\partial_y+\frac{4\,x\,\partial_x}{r+2} \end{array}\right],[n=-3]\right],\left[\begin{array}{c}0\end{array}\right]$$

$$7=\left[\left[\begin{array}{c} x\,\partial_x \end{array}\right],[n=-3,r=-2]\right],\left[\begin{array}{c}0\end{array}\right]$$

$$8=\left[\left[\begin{array}{c} y\,\partial_y+2\,x\,\partial_x \\ \partial_x \\ y\,x\,\partial_y+x^2\,\partial_x \end{array}\right],[n=-3,r=0]\right],\left[\begin{array}{ccc} 0 & -2\,\partial_x & 2\,y\,x\,\partial_y+2\,x^2\,\partial_x \\ 2\,\partial_x & 0 & y\,\partial_y+2\,x\,\partial_x \\ -2\,y\,x\,\partial_y-2\,x^2\,\partial_x & -y\,\partial_y-2\,x\,\partial_x & 0 \end{array}\right]$$

$$9=\left[\left[\begin{array}{c} y\,\partial_y-\frac{x\,\partial_x}{r+2} \end{array}\right],[n=2]\right],\left[\begin{array}{c}0\end{array}\right]$$

$$10=\left[\left[\begin{array}{c} x\,\partial_x \end{array}\right],[n=2,r=-2]\right],\left[\begin{array}{c}0\end{array}\right] \qquad \textbf{(4)}$$

For case 11 we have 4 subcases.

```
> eq[11][Constraint];
```

$$\left[1500\,r+1525\,r^2+490\,r^3+49\,r^4=0\right] \qquad \textbf{(5)}$$

```
> solve(%,r);
```

$$\{r=0\},\left\{r=-\frac{15}{7}\right\},\left\{r=-\frac{20}{7}\right\},\{r=-5\} \qquad \textbf{(6)}$$

```
> for i in % do i=G[11][op(i)],Commutator(G[11][op(i)][1]) od;
```

$$\{r=0\}=\left[\left[\begin{array}{c} \partial_x \\ y\,\partial_y-\frac{1}{2}\,x\,\partial_x \end{array}\right],[n=2]\right],\left[\begin{array}{cc} 0 & -\frac{1}{2}\,\partial_x \\ \frac{1}{2}\,\partial_x & 0 \end{array}\right]$$

$$\left\{r = -\frac{15}{7}\right\} = \left[\left[\begin{array}{c} \frac{1}{7}\, y\, \partial_y + x\, \partial_x \\ -\frac{3}{343}\, \frac{\left(4\, x^{1/7} - 49\, a\, y\right)\, \partial_y}{a\, x^{1/7}} + x^{6/7}\, \partial_x \end{array}\right], [n = 2]\right],$$

$$\left[\begin{array}{cc} 0 & \frac{3}{2401}\, \frac{\left(4\, x^{1/7} - 49\, a\, y\right)\, \partial_y}{a\, x^{1/7}} - \frac{1}{7}\, x^{6/7}\, \partial_x \\ -\frac{3}{2401}\, \frac{\left(4\, x^{1/7} - 49\, a\, y\right)\, \partial_y}{a\, x^{1/7}} + \frac{1}{7}\, x^{6/7}\, \partial_x & 0 \end{array}\right]$$

$$\left\{r = -\frac{20}{7}\right\} = \left[\left[\begin{array}{c} \frac{6}{7}\, y\, \partial_y + x\, \partial_x \\ \frac{4}{343}\, \frac{\left(49\, a\, x^{1/7}\, y + 3\, x\right)\, \partial_y}{a} + x^{8/7}\, \partial_x \end{array}\right], [n = 2]\right],$$

$$\left[\left[0, \frac{4}{2401}\, \frac{\left(49\, a\, x^{1/7}\, y + 3\, x\right)\, \partial_y}{a} + \frac{1}{7}\, x^{8/7}\, \partial_x\right], \left[-\frac{4}{2401}\, \frac{\left(49\, a\, x^{1/7}\, y + 3\, x\right)\, \partial_y}{a} - \frac{1}{7}\, x^{8/7}\, \partial_x, 0\right]\right]$$

$$\{r = -5\} = \left[\left[\begin{array}{c} y\, \partial_y + \frac{1}{3}\, x\, \partial_x \\ y\, x\, \partial_y + x^2\, \partial_x \end{array}\right], [n = 2]\right], \left[\begin{array}{cc} 0 & \frac{1}{3}\, y\, x\, \partial_y + \frac{1}{3}\, x^2\, \partial_x \\ -\frac{1}{3}\, y\, x\, \partial_y - \frac{1}{3}\, x^2\, \partial_x & 0 \end{array}\right] \quad (7)$$