Radial non-linear wave equation

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> with(Exterior):
```

Exterior calculus package, version 1.12 (30 Oct 2009).
 (1)

```
> depend([u],F);
```

$$F \qquad (2)$$

```
> deq:=Diff(u,t$2)=Diff(u,r$2)+m*Diff(u,r)/r+F;
```

$$deq := \frac{\partial^2}{\partial t^2} u = \frac{\partial^2}{\partial r^2} u + \frac{m \left(\frac{\partial}{\partial r} u\right)}{r} + F \qquad (3)$$

The constraint option removes the linear cases.

```
> symmetry,eq:=find_symmetry(deq,Constraints={diff(F,u$2)<>0,m<>0}
,casesplit):
```

```
> caseplot(eq,pivots);
```

========= Pivots Legend =========

$$p1 = \left(\frac{\partial^3}{\partial u^3} F\right) m^2 - 4 r^2 \left(\frac{\partial^3}{\partial u^3} F\right) \left(\frac{\partial}{\partial u} F\right) + 4 \left(\frac{\partial^2}{\partial u^2} F\right)^2 r^2 - 2 \left(\frac{\partial^3}{\partial u^3} F\right) m$$

$$\begin{aligned} p2 = {} & -2 \left(\frac{\partial^3}{\partial u^3} F\right)^2 m^2 + \left(\frac{\partial^2}{\partial u^2} F\right) m^2 \left(\frac{\partial^4}{\partial u^4} F\right) + 8 \left(\frac{\partial^3}{\partial u^3} F\right)^2 m \\ & - 6 \left(\frac{\partial^2}{\partial u^2} F\right) m \left(\frac{\partial^4}{\partial u^4} F\right) - 8 \left(\frac{\partial^3}{\partial u^3} F\right)^2 + 8 \left(\frac{\partial^3}{\partial u^3} F\right)^2 r^2 \left(\frac{\partial}{\partial u} F\right) \\ & - 4 \left(\frac{\partial^2}{\partial u^2} F\right)^2 r^2 \left(\frac{\partial^3}{\partial u^3} F\right) + 8 \left(\frac{\partial^2}{\partial u^2} F\right) \left(\frac{\partial^4}{\partial u^4} F\right) \\ & - 4 \left(\frac{\partial^2}{\partial u^2} F\right) r^2 \left(\frac{\partial^4}{\partial u^4} F\right) \left(\frac{\partial}{\partial u} F\right) \end{aligned}$$

$$p3 = \left(\frac{\partial}{\partial u} F\right) F \left(\frac{\partial^3}{\partial u^3} F\right) + \left(\frac{\partial^2}{\partial u^2} F\right) \left(\frac{\partial}{\partial u} F\right)^2 - 2 \left(\frac{\partial^2}{\partial u^2} F\right)^2 F$$

$$p4 = \left(\frac{\partial^3}{\partial u^3} F\right) (m - 2)$$

$$p5 = \left(\frac{\partial^2}{\partial u^2} F\right) F m + 4 \left(\frac{\partial^2}{\partial u^2} F\right) F - 4 \left(\frac{\partial}{\partial u} F\right)^2$$

$$p6 = m - 2$$

$$p7 = m^2 \left(\frac{\partial}{\partial u} F\right) - 18 \left(\frac{\partial}{\partial u} F\right) m + 8 F \left(\frac{\partial^2}{\partial u^2} F\right) r^2 + 48 \left(\frac{\partial}{\partial u} F\right) - 4 r^2 \left(\frac{\partial}{\partial u} F\right)^2$$

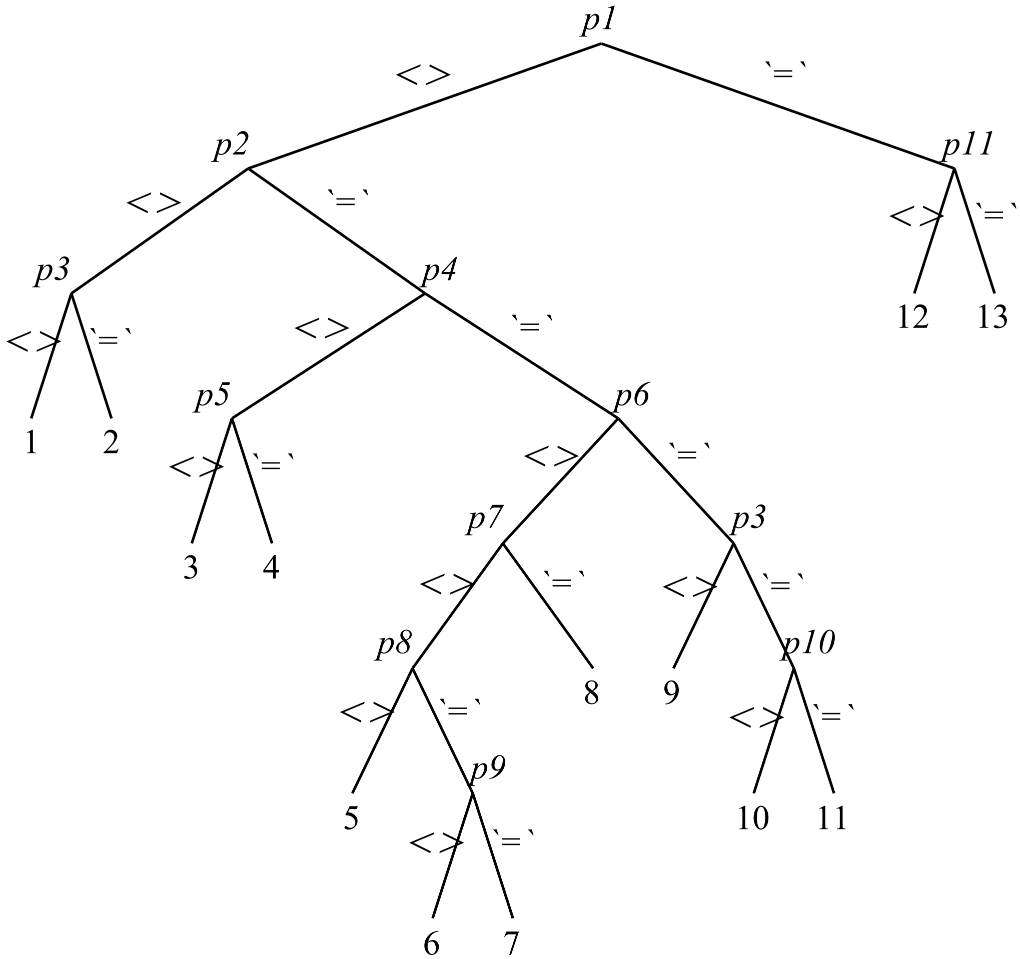
$$p8 = -\left(\frac{\partial}{\partial u} F\right)^2 + 2 \left(\frac{\partial^2}{\partial u^2} F\right) F$$

$$p9 = m - 4$$

$$p10 = 3 \left(\frac{\partial^2}{\partial u^2} F\right) F - 2 \left(\frac{\partial}{\partial u} F\right)^2$$

$$p11 = \left(\frac{\partial^2}{\partial u^2} F\right) F - \left(\frac{\partial}{\partial u} F\right)^2$$

Rif Case Tree




```
> soln:=rifsolve(eq,Parameters={F}):
> G:=one_parameter(symmetry,soln):
> for i from 1 to soln[casecount] do print(i,G[i]) od;
```

$$1, \left[\left[\, \partial_t \,\right]\right]$$

$$2, \left[\left[\begin{matrix} \partial_t \\ \left(\frac{1}{2}\, t\_A1 - \frac{1}{2}\, t\right) \partial_t + \frac{1}{2}\, (\_A1 - 1)\, r\, \partial_r + (-u + \_A2)\, \partial_u \end{matrix}\right], \left[F = (-u + \_A2)^{-\_A1}\, \_A3\right]\right]$$

$$3, \left[\left[\partial_t \right], \left[F = \frac{1}{m+4} \left(m\_A1\, 256^{-\frac{1}{m}} \left(\frac{1}{m\,(u+\_A2)} \right)^{-\frac{4}{m}} u + m\_A1\, 256^{-\frac{1}{m}} \left(\frac{1}{m\,(u+\_A2)} \right)^{-\frac{4}{m}} \_A2 + \_A3\, m + 4\,\_A3 \right) \right] \right]$$

$$4, \left[\left[\begin{matrix} \partial_t \\ t\,\partial_t + r\,\partial_r - \frac{1}{2}\, \frac{m\,(\_A1\, u + \_A2)\,\partial_u}{\_A1} \\ (t^2+r^2)\,\partial_t + 2\,t\,r\,\partial_r - \frac{t\,m\,(\_A1\, u + \_A2)\,\partial_u}{\_A1} \end{matrix} \right], \left[F = \frac{\left(\frac{m+4}{m\,(\_A1\, u + \_A2)} \right)^{-\frac{4}{m}} m\,(\_A1\, u + \_A2)}{m+4} \right] \right]$$

$$5, \left[\left[\partial_t \right], \left[F = \frac{1}{2}\,\_A1\, u^2 + \_A2\, u + \_A3 \right] \right]$$

$$6, \left[\left[\begin{matrix} \partial_t \\ -\frac{1}{2}\, t\,\_A1\, \partial_t - \frac{1}{2}\, r\,\_A1\, \partial_r + (\_A1\, u + \_A2)\,\partial_u \end{matrix} \right], \left[F = \frac{1}{4}\,\_A1^2\, u^2 + \frac{1}{2}\,\_A1\, u\,\_A2 + \frac{1}{4}\,\_A2^2 \right] \right]$$

$$7, \left[\left[\begin{matrix} \partial_t \\ t\,\partial_t + r\,\partial_r - \frac{2\,(\_A1\, u + \_A2)\,\partial_u}{\_A1} \\ (t^2+r^2)\,\partial_t + 2\,t\,r\,\partial_r - \frac{4\,t\,(\_A1\, u + \_A2)\,\partial_u}{\_A1} \end{matrix} \right], \left[F = \frac{1}{4}\,\_A1^2\, u^2 + \frac{1}{2}\,\_A1\, u\,\_A2 \right.$$

$$+\frac{1}{4}\_A2^2, m=4\Big]\Big]$$

$$8, table\Bigg(\Bigg[\Big(m=9+\sqrt{33}\Big)=\Bigg[\Bigg[\begin{matrix}\partial_t\\ -\frac{1}{2}t\_A1\,\partial_t-\frac{1}{2}r\_A1\,\partial_r+(\_A1\,u+\_A2)\,\partial_u\end{matrix}\Bigg],\Big[F=\frac{1}{4}\_A1^2u^2+\frac{1}{2}\_A1\,u\_A2+\frac{1}{4}\_A2^2\Big]\Bigg],\Big(m=9-\sqrt{33}\Big)=\Bigg[\Bigg[\begin{matrix}\partial_t\\ -\frac{1}{2}t\_A1\,\partial_t-\frac{1}{2}r\_A1\,\partial_r+(\_A1\,u+\_A2)\,\partial_u\end{matrix}\Bigg],\Big[F=\frac{1}{4}\_A1^2u^2+\frac{1}{2}\_A1\,u\_A2+\frac{1}{4}\_A2^2\Big]\Bigg]\Bigg]\Bigg)$$

$$9, \Bigg[\Big[\partial_t\Big], \Bigg[F=-\frac{\left(-\frac{\_A1}{u+\_A3}\right)^{\_A1}\_A2\,u+\left(-\frac{\_A1}{u+\_A3}\right)^{\_A1}\_A2\,\_A3-\_A4\,\_A1+\_A4}{\_A1-1}, m=2\Bigg]\Bigg]$$

$$10, \Bigg[\Bigg[\begin{matrix}\partial_t\\ \left(\frac{1}{2}t\_A1-\frac{1}{2}t\right)\partial_t+\frac{1}{2}(\_A1-1)\,r\,\partial_r+(-u+\_A2)\,\partial_u\end{matrix}\Bigg], \Big[F=(-u+\_A2)^{\_A1}\_A3, m=2\Big]\Bigg]$$

$$11, \left[\left[\begin{array}{c} \partial_t \\ t\,\partial_t + r\,\partial_r - \frac{(\_A1\,u + \_A2)\,\partial_u}{\_A1} \\ (t^2 + r^2)\,\partial_t + 2\,t\,r\,\partial_r - \frac{2\,t\,(\_A1\,u + \_A2)\,\partial_u}{\_A1} \end{array}\right], \left[F = \frac{1}{27}\,\_A1^3\,u^3 + \frac{1}{9}\,\_A1^2\,u^2\,\_A2 + \frac{1}{9}\,\_A1\,u\,\_A2^2 + \frac{1}{27}\,\_A2^3, m = 2\right]\right]$$

$$12, \left[\left[\,\partial_t\,\right], \left[F = \frac{\mathrm{e}^{\_A1\,(u + \_A2)} + \_A3\,\_A1}{\_A1}, m = 2\right]\right]$$

$$13, \left[\left[\begin{array}{c} \partial_t \\ -\frac{1}{2}\,t\,\_A1\,\partial_t - \frac{1}{2}\,r\,\_A1\,\partial_r + \partial_u \end{array}\right], \left[F = \mathrm{e}^{\_A1\,u}\,\_A2, m = 2\right]\right] \qquad \textbf{(4)}$$

Case 8 has subcases. Note that the cases are based on pivots and so are not necessarily distinct.