Conference on Finite Groups and Representations

List of abstracts

The McKay Conjecture for Some Finite Groups

Jianbei An (Auckland)

The McKay conjecture states that the number of irreducible complex characters of a finite group of degree coprime with a fixed prime p is equal to the same number for the normalizer of a Sylow p-subgroup. The conjecture recently has been reduced by Isaacs-Malle-Navarro to the simple groups. Four open problems have been raised by Malle. In this talk we'll discuss some of the problems.

Finding involutions in the Big Ree groups

Henrik Bäärnhielm (Auckland)

Our interest is in the Big Ree groups ${}^2F_4(q)$, with q an odd power of 2, one of the twisted families of finite simple groups. We want to be able to do constructive membership testing and constructive recognition, given a set X of generators for the group, and this can be done if we can find involutions, as straight line programs in X.

I will describe a practical method for doing this in the natural representation, which has dimension 26. It will be a polynomial time algorithm except for the use of discrete log.

Complete Reducibility and Separability

Michael Bate (Oxford)

Serre's notion of G-complete reducibility for subgroups of a reductive group G has its roots in representation theory. However, it can also be interpreted in a geo-

metric setting, allowing us to apply techniques from Geometric Invariant Theory (GIT). In this talk I will outline the main ideas involved and give examples of how they have been applied in joint work with B.Martin, G. Roehrle and R. Tange. I will also describe how we have recently used ideas from G-complete reducibility to motivate new GIT results.

Algorithms for Modules and Matrix Algebras

Peter Brooksbank (Bucknell)

In this talk I will discuss some fundamental computational problems concerning finitely generated algebras and their modules. The focus will be on algorithms that work in the *arithmetic model*, wherein the fundamental steps are basic field operations. In particular, I will present new deterministic techniques for finding certain direct sum decompositions of modules. These techniques have applications to the problem of testing for isomorphism between two given modules, and to the determination of the socle of a module. This is principally a report on recent joint work with Gene Luks (University of Oregon).

Hyperbolic Kac-Moody Groups

Lisa Carbone (Rutgers)

Let K be a Kac-Moody Lie algebra and let G denote a Kac-Moody group associated with K. When K is of affine type, the commutator subalgebra of K is a central extension of a loop algebra and G is a central extension of a classical group. However if K is of hyperbolic type, many fundamental questions about the structure of K and G remain unanswered. We outline a program to investigate the structure of G when K is of hyperbolic type, focusing on the case where G is constructed over a finite field. In this case G is a locally compact group acting on a hyperbolic Tits building, and finite groups can be used to generate lattice subgroups with intricate arithmetic structure.

Connectedness of generating sets for finite groups

Marston Conder (Auckland)

Suppose G is a finitely-generated group, and X and Y are two generating sets for G with $|X| = |Y|$. Under what conditions can X be 'transformed' to Y by a sequence of Nielsen transformations? or other single-element replacements? Some aspects of these questions will be described, and recent results given in the case of

generating pairs for 2-generator finite groups.

Groups with the same categories of representation

Alexei Davydov (Macquarie)

A (quasi)-twisted isomorphism of group algebras is an algebraic relation, representing isomorphisms of character tables of finite groups. Related notions and examples will be discussed.

On Modular Invariant Theory of Finite Groups

Peter Fleischmann (Kent)

Let k be a field and G a finite group, acting on the polynomial ring $A := k[x_1, \ldots, x_d]$ by graded k-algebra automorphisms. The ring of invariants $A^G := \{f \in A \mid g(f) = f\}$ is the main object of study in Invariant Theory. The theory is very well developed in the "classical case", where the characteristic of k is zero, but far less so in the case of positive characteristic p, in particular the "modular case", where p divides the group order $|G|$.

In that case there are open questions about the constructive complexity of A^G, measured by degree bounds for generators, and about the structural complexity, measured by the depth (=cohomological co-dimension) of A^G as a module over a homogeneous system of parameters.

I will report on some recent results dealing with both types of questions. They include the recent solution of the degree bound problem for modular invariant rings of arbitrary finite-dimensional $\mathbb{Z}/p\mathbb{Z}$-representations, and a new general algorithm to construct A^G, which grew out of the study of certain localisations of invariant rings. These localisations also lead into the interesting subject of commutative, projective $k-G$ algebras and their invariants, which have a particularly nice structure theory in the case of p-groups in characteristic p.

Unipotent conjugacy in finite groups of Lie type

Simon Goodwin (Birmingham)

Let p be a prime and q a power of p. Let G be a connected split reductive algebraic group defined over $\mathbb{F}_p$ and write $G(q)$ for the finite group of $\mathbb{F}_q$-rational points of G. Let P be a parabolic subgroup of G defined over $\mathbb{F}_p$ with unipotent radical U. In this talk we will discuss results regarding parameterizations of (and the number of): conjugacy classes of $U(q)$, and $U(q)$-conjugacy classes in all of $G(q)$.

Subfactors as a generalisation of groups

Vaughan Jones (Berkeley)

The hyperfinite II_1 factor is a universal object for finite groups – each one acts in a unique way by outer automorphisms. And the fixed point subfactor remembers the group in a Galois-like way. Thus general subfactors are a generalisation of finite groups. We will examine such ideas as transitivity of group actions in this context.

Unipotent classes in classical groups over fields of characteristic 2

Martin Liebeck (Imperial)

The theory of unipotent conjugacy classes in classical algebraic groups is well known and fairly elementary when the underlying characteristic p is either zero or a good prime, but is far less well understood when the characteristic p is a bad prime, i.e., when $p = 2$ and the group is a symplectic or orthogonal group. In the talk I shall present an approach which gives all the basic information on unipotent classes in these groups: class representatives, centralizer dimensions and structure, corresponding information for finite classical groups, and a relationship with nilpotent orbits in the Lie algebra of the group.

Arithmetic hyperbolic groups

Gaven Martin (Massey)

We report on our programme to identify the finitely many arithmetic hyperbolic generalised triangle groups. Among other things we establish a conjecture of Hilden, Lozano and Montesinos concerning the arithmetic $(n, 0)$ surgeries on two bridge link complements, and also the following unexpected result: There are no co-

compact arithmetic generalised triangle groups with generators of orders at least 6.

Cyclotomic Solomon algebras

Andrew Mathas (Sydney)

We will discuss an analogue of the Solomon descent algebra for the complex reflection groups of type $G(r, 1, n)$. As with the Solomon descent algebra, our algebra has a basis given by sums of 'distinguished' coset representatives for certain 'reflection subgroups'. We explicitly describe the structure constants with respect to this basis and show that they are polynomials in r. This allows us to define a deformation, or q-analogue, of these algebras which depends on a parameter q. We determine the irreducible representations of all of these algebras and give a basis for their radicals. Finally, we show that the direct sum of cyclotomic Solomon algebras is canonically isomorphic to a concatenation Hopf algebra.

This is joint work with Rosa Orellana

Constructive recognition for classical groups

Eamonn O'Brien (Auckland)

Algorithms for the study of classical groups rely on the availability of an effective isomorphism between a classical group and its natural representation. We report on recent work which exploits involution centralisers to construct such isomorphisms.

Complete Reducibility and Separability

Gerhard Röhrle (Bochum)

Let G be a reductive linear algebraic group over an algebraically closed field of characteristic $p > 0$. A subgroup of G is said to be separable in G if its global and infinitesimal centralizers have the same dimension. We study the interaction between the notion of separability and Serre's concept of G-complete reducibility for subgroups of G. The separability hypothesis appears in many general theorems concerning G-complete reducibility. We prove that if G is a connected reductive group and p is very good for G, then any subgroup of G is separable; we deduce that under these hypotheses on G, a subgroup H of G is G-completely reducible provided the Lie algebra of G is semisimple as an H-module. This is a report on

joint work with M. Bate, B. Martin, and R. Tange.

Unipotent and nilpotent elements in algebraic groups and associated Lie algebras

Gary M. Seitz (Oregon)

In this talk I will describe a joint project with Martin Liebeck aimed at describing the conjugacy classes and centralizers of unipotent and nilpotent elements in simple algebraic groups. The goal is to obtain precise results for all types of simple groups and in all characteristics. I will begin with the basic outline of our approach and this will be followed by some results on nilpotent classes in classical groups.

A double centraliser theorem for the symplectic group

Rudolf Tange (Bochum)

I will discuss a double centraliser theorem for the symplectic group in positive characteristic. This is a symplectic version of well known double centraliser property of the general linear and the symmetric group.

As in the case of the general linear group there is a relation between the two representation theories. I will indicate what this relation is in the case of the symplectic group and the Brauer algebra. This is still subject of ongoing research.