Modelling Invasive Species and Weed impact

NZIMA Funded University of Canterbury Research Programme 2007 - 2009

Case Study 1 – A framework for weed management

Aims
The first case study for the key workshop will form the basis of the project for the post-doctoral fellowship. The key aims of the fellowship will be to:

- Formulate a simple deterministic model of weed spread and control.
- Develop a stochastic model from the deterministic framework and examine the effects of stochasticity on the invasion dynamics.
- Identify optimal management strategies.

Using a deterministic approach, the arrival of a species into an area and its subsequent growth and spread in a structured environment can be modelled using either mean field models or reaction–diffusion equations. The costs of detection and control at various points in the species life cycle can be incorporated to assess potential control strategies. However, when the underlying events governing species emergence are rare and the rate of invasion is subject to environmental variability (as is the case for most exotic plant species), a more realistic method is to model the system stochastically. This allows for random fluctuations in the environment and in the rates of weed emergence and spread. This method also has the advantage of being able to include effects due to clustering of species and spatial heterogeneity.

Method
The spread of plant species can be modelled deterministically using ordinary or partial differential equations (Williamson, 1996). For example, an SIS model (Kermack and McKendrick, 1927)

$$\frac{dS}{dt} = -\beta SI - \alpha S + \gamma I$$

$$\frac{dI}{dt} = \beta SI + \alpha S - \gamma I$$

can be used to model the total amount of 'susceptible' (S) and 'infected' (I) land. Similarly, the Fisher (1937) equation

$$\frac{\partial u}{\partial t} = ru(1-u) + D\nabla^2 u$$

for population density (u) represents a classical nonlinear reaction–diffusion model of species growth and spread. These simple differential equations can be extended to multi-compartment models for the case of several plant species.

Deterministic models of this type can used to predict macroscopic rates of species invasion in a steady, homogeneous environment (Woolcock and Cousens, 2000). Simple functions may be included to simulate the effects of various control strategies. These might include reducing the intrinsic growth rate, reducing the species 'diffusion coefficient' or increasing the death rate. In addition, a cost function can be specified, quantifying the aggregate environmental and economical cost of a particular level of weed invasion together with a particular monitoring and control protocol.

The post-doctoral fellow will formulate a simple deterministic framework of this type for consideration at the key workshop. Following detailed discussion and development at the workshop, and in collaboration with workshop participants, the post-doctoral fellow will then extend the basic framework to a stochastic model. This could be done in a number of different ways. The emergence of new weed species and arrival of existing weeds at new sites could be modelled as stochastic processes (*e.g.* Poisson process). The physical spread of a weed species could be modelled as a two-dimensional random walk, which would allow the effect of a heterogeneous environment to be studied (Law *et al*, 2003). This aspect of the model can be approached in two ways: as a stochastic individual-based model; or using a continuum limit PDE. The link between these two approaches has been studied for relatively simple reinforced random walks and non-interacting populations (Othmer and Stevens, 1997), but is still an open problem for more general walks and for interacting populations. Also, the question of whether a species spreads contiguously as a single advancing front or by forming new isolated colonies will have major implications for selecting appropriate monitoring and management methods (Wallinga *et al*, 2002).

The superposition of control strategies and cost functions on the stochastic model poses new mathematical questions. In particular, a simple stochastic model might be expected to give the same behaviour, on average, as its mean field equivalent (Øksendal, 2003). However, Jensen's inequality states that the mean of a nonlinear function differs from the function of a mean. Thus, even when the expected solution of a stochastic model is the same as that of the deterministic skeleton, the application of a control and/or cost function can radically alter the outcome (Pitchford *et al*, 2005). Addressing the implications of this phenomenon for the selection of optimal control strategies will form a central part of this project.

Many existing models implicitly assume that there is no interaction among different species or between the plants and the environment (Hastings, 1997). The addition of species–species and species–environment interactions is therefore a priority and will result in a much more realistic model. In general these interactions will be nonlinear in nature, resulting in complex and often unexpected behaviour rather than simply a sum of linear effects.

One area of weed management that has been the subject of mathematical modelling is the identification of the minimum threshold density of weeds that justifies initiation of an active control mechanism (Moore, 1989; Doyle, 1997). These models are generally non-spatial in that they assume a uniform distribution of weeds. Furthermore, the existence of uncertainty in the cost and control response functions, as well as environmental fluctuations, could substantially alter the predicted threshold density. This has been identified as a major limitation of existing models (Doyle, 1997) and a stochastic model would provide the ideal framework for studying the effects of uncertainty.

Proposed Time Line for the Post-Doctoral Fellow

The postdoctoral fellowship will start in June 2006. The fellow will begin by developing a simple deterministic model of weed management that forms the basis of the first case study at the key workshop in Dec 2006. Using the ideas generated at the workshop the fellow will then formulate the stochastic model and explore other avenues generated by the workshop. In Sept 2007 the analysis and results to that point will be presented at the follow-up meeting.

Personnel

The construction and analysis of the models will require an imaginative and technically competent scientist. The aspects of ecology and general modelling that are pertinent to this project can, with appropriate support and enthusiasm, be learnt. An interest in ecological systems is, therefore, a necessary requirement, but an in-depth knowledge, whilst desirable, is not necessary. The scientist must be competent in the mathematical aspects, particularly the techniques of stochastic processes, beforehand. For this a degree and PhD in mathematics, statistics or another suitably numerate discipline are required. An emphasis, at graduate level study, in either mathematical or statistical modelling is also highly desirable. The candidate must be competent in using computational methods as it is anticipated that a significant proportion of the analysis will be done by computational means. Finally, good communication skills and written English are necessary in order to undertake collaborations with scientists from more experimental backgrounds and to present the work to a wider audience. The post-doctoral fellow will benefit from input from both mathematicians and statisticians at Canterbury and ecologists at Lincoln and Landcare.

Outputs

The research is expected to generate published work in both mathematical and ecological journals. Results will also be disseminated at national and international conferences, *e.g.* the Annual Meeting of the Society for Mathematical Biology. The post-doctoral fellow will be expected to play a key role in all outputs of the research, from writing and co-authoring journal articles to presenting the work at scientific meetings. The project will also generate outputs in the form of advice to weed management agencies, such as DoC, and by feeding into the FRST-funded project at Landcare Research. Using these outputs, during the second year of the research the project leaders will apply for funding from external sources (*e.g.* FRST) to continue work in this area.

The post-doctoral fellow will be based in the Department of Mathematics and Statistics at the University of Canterbury. He/she will benefit from close collaborations with theoretical ecologists at Lincoln University and Landcare Research, who have recently negotiated a twelve-year funding contract from FRST on Ecosystem Resilience to Weeds.

On conclusion of the project, the post-doctoral fellow will write a report on the project, its outputs and the progress made to be submitted for publication and distributed to programme participants.

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