Speakers, Titles and Abstracts

July 21, 2010

Catalyzed by the fundamental work of Poincaré, the twentieth century saw topology develop into the mathematician's primary tool for the global analysis of nonlinear spaces and functions. Because of modern information technology, which provides us with fast computers and the ability to collect and manipulate large quantities of data, the beginning of the twenty-first century is witnessing the application of these tools to data analysis, image processing, and computational dynamics. The goal of this workshop is to present a coherent (though incomplete) *introduction* to these new methods. While everyone interested in the subject is invited to attend, the lectures are aimed at upper level undergraduates and beginning graduate students. The first few lectures will provide basic introductions to dynamics and algebraic topology and the later lectures will build on these ideas to demonstrate how they can be used both theoretically and computationally.

1. Hiroshi Kokubu, **Dynamical systems: a basic introduction**

   Abstract: Some basic notions of dynamical systems are discussed together with examples. Within the given lecture time, I will try to cover as many topics as possible from the following: definition of the dynamical system, invariant set, stability, recurrence, robustness and hyperbolicity, bifurcation, etc.

2. Hiroe Oka, **Dynamics of globally coupled maps**

   Abstract: A system of globally coupled maps (abbreviated as GCM) was studied by K. Kaneko as one of the simplest examples of complex systems that exhibit interesting collective behaviors such as synchronization, so-called "chaotic itinerancy", etc. In this lecture, some computational results mainly obtained by M. Komuro are reviewed. An attempt to understand the "chaotic itinerancy" will also be discussed.

3. Tomas Gedeon, **Modeling dynamics of gene regulation**

   Abstract: We will introduce some biological models arising in cellular biology and gene regulation. We will discuss model selection between stochastic and deterministic dynamics as well as selection of nonlinearities in deterministic models. In view of parametric and experimental uncertainty, we will emphasize that the analysis of biological models requires robust methods, the conclusions of which are valid for a range of parameters and nonlinearities.

4. Marian Mrozek, **Homology: a classical topological invariant in algorithmic perspective**

   Abstract: Homology groups have been invented in early XX century as a topological tool motivated by some problems of differential equations. In fact, certain difficult existence problems in the theory of differential equations, in particular the existence of chaotic dynamics, can be reduced to questions about the homology of some sets in R^n and continuous maps acting on them. The sets and some information about the maps may be derived algorithmically via rigorous numerical enclosures of the trajectories of the system based on interval arithmetic. The problem is that although the topology of the sets is in general simple, the sets are often huge. For many problems the classical homology algorithms are not fast enough for the computer-assisted proofs to succeed. Also, there are no standard algorithms which can find the homology maps on the basis of the information available from the rigorous numerics of differential equations. However, some non-standard techniques based on reduction algorithms and cubical homology of multivalued maps make such computations possible.

   Part I

   (a) Topological spaces, homotopy equivalence.
   (b) Discretization of topological spaces: simplicial and cubical sets
   (c) Chains and chain maps
   (d) Homology groups, homology functor
   (e) The classical algorithm for homology groups
   (f) Limitations of classical approach

5. Zin Arai, **Uniform hyperbolicity and structural stability of dynamical systems: from a computational viewpoint: Part I**

   Abstract: In this talk, we discuss how to verify the uniform hyperbolicity and structural stability of given dynamical systems. We first recall the definition and the motivation of hyperbolicity and stability, and then discuss the close relation between them. The importance of these notions will be explained using examples such as Smale horseshoe and the Henon map. To design a rigorous and efficient algorithm for proving uniform hyperbolicity, we introduce several topological and computational methods; the construction of the dynamics induced on the tangent bundle and the projectivized tangent bundle, the Morse decomposition of dynamical systems, and some graph theoretical algorithms.

6. Tomas Gedeon, **Introduction to Conley index theory**

   Abstract: We will introduce Conley index theory as a robust tool of analysis of continuous and discrete time dynamical systems. The emphasis will be on continuous time dynamics where Conley index has clearer geometrical interpretation. We will show how Conley index can be used to show existence of an equilibrium, periodic orbit and if time permits, connecting orbits.

7. Marian Mrozek, **Homology: a classical topological invariant in algorithmic perspective**

   Abstract: Part II

   (a) Algebraic reductions
   (b) Reductions via acyclic subspace
   (c) Free face reductions, coreduction algorithm
   (d) Reductions via a discrete Morse theory
   If time permits:
   (e) Multivalued representations of continuous maps
   (f) Computing homology of maps via chain selectors and graph projections

8. Zin Arai, **Uniform hyperbolicity and structural stability of dynamical systems: from a computational viewpoint: Part II**

9. Hiroshi Kokubu, **Dynamics of Hebbian models of associative memory**

   Abstract: Hebbian models of associative memory are some kind of neural networks which represent a certain brain function that appears when one tries to recall something. There have been a lot of results obtained by 1990's for these models in the case of large number of neurons, by using methods from statistical physics. Here I consider such (noiseless) Hebbian neural network models with a rather small number of neurons, and discuss how a recent method in computational dynamics may be used to reveal the phase space structure of such "dynamical systems".

10. Tomas Kaczynski, **Singular zones in discrete data sets**

   Abstract: The mathematical tool commonly used in computer vision to classify critical points of functions is the Morse theory. This theory relies on an assumption that the model function f is smooth and "generic", that is, its critical points (the zeroes of the gradient of f) are isolated and have distinct critical values. The smoothness may be simulated on discrete data via an interpolation by a generic function. This approach is natural in many geometric modeling problems but not suitable in others. I will give a brief overview of the Morse theory, and then present a recently

developed approach, inspired by concepts from the Conley index theory, which provides a discrete model of non-isolated singularities in discrete data sets of arbitrary dimension.

11. Konstantin Mischaikow, **Topological analysis of spatial-temporal data**

    Abstract: Many physical phenomena are interesting precisely because they produce complicated time dependent patterns. I will discuss how computational homology can be used to quantify these patterns. I will also discuss the question of the validity of these methods.