You may have heard of Lorenz’s famous question: “Does the flap of a butterfly’s wings in Brazil set off a tornado in Texas?”. The question was meant to portray one of the main concepts in chaos theory (sensitive dependence to initial conditions). This course will teach theory, techniques and applications of systems of nonlinear equations. In all cases, we are concerned with dynamics: the time evolution of spatial structure. We will cover the mathematics behind chaos theory, and learn techniques for analysing nonlinear systems. Since it is usually difficult or impossible to write down an exact solution to systems of nonlinear equations, the emphasis will be on qualitative techniques for classifying and understanding the behaviour of nonlinear systems. Both main types of dynamical system will be studied: discrete systems, consisting of an iterated map; and continuous systems, arising from nonlinear differential equations. The natural relationships between discrete and continuous time systems will be emphasised too.

This course is independent of Math363 Dynamical systems, although previous enrolment there is desirable.

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Class times: Wed, Thurs, Fri 9-10am, Erskine 505. The Wed+Thurs classes are lectures, tutorial on Friday.

Homework and assessment: Problem sheets will be given out approximately fortnightly. There will be three sets of hand-in problems worth 10% each (due 4pm Wednesday 25/3, 6/5, 27/5). A project worth 10% (essay based on reading a paper, and oral presentation) will be due on 3/6. A three hour final exam worth 60% will be scheduled in the usual end of Semester exam period. Assignments should be done individually, but the project can be done in pairs.

Syllabus

- Flows as dynamical systems.
- Hyperbolicity, linearisation and stability of fixed points and period orbits.
- Invariant manifolds and phase portrait analysis.
- Centre manifolds and local bifurcations; global bifurcations.
- Maps as dynamical systems (including return maps).
- Chaotic behaviour of one-dimensional maps, including period-doubling.
- Symbolic dynamics.
- Topic(s) selected from: fast-slow systems; Lyapunov exponents, entropy and randomness; non-autonomous dynamics.
Recommended reading

The recommended books for this course are the texts by Alligood, Sauer and Yorke [1] and Meiss [7]. The Math363 text by Strogatz [9] is also an excellent supplement, and source of background material.

Lorenz equations: the development of the modern theory of dynamical systems has its roots in Lorenz’s famous work on simplified models of atmospheric circulation. The equations which bear his name appear in the original paper [6], and subsequent decades have seen careful and beautiful unravelling of the delicate mechanisms and geometry causing the chaotic phenomena observed therein. The underlying structures were correctly identified in the 1970s by Williams and Guckenheimer (amongst others), and these ideas — interlaced with dynamical systems theory — are presented in the classic text of Guckenheimer & Holmes [4] (particularly chapters 2, 5 and 6). A comprehensive numerical and geometric study was done by Colin Sparrow in his PhD thesis, and published as [8]. More recently, computer power has advanced to such an extent that the well-founded geometric conjectures of earlier decades have been numerically proven: the proof of chaos in the “Lorenz attractor” was completed by Warwick Tucker (Erskine visitor to UC in 2011) in his PhD thesis, and published in [10]. The 2006 paper of Doedel, Krauskopf and Osinga [3] also outlines the source of the chaos in the Lorenz system, and describes (with amazing pictures) numerical computations of the “Lorenz manifold” which organises the phase space (both Krauskopf and Osinga are professors at the University of Auckland).

Also recommended: Alan Turing’s article [11], which essentially invents the theory of pattern formation.


References