Newsletter
Mathematics & Statistics

UNIVERSITY OF CANTERBURY
Te Whare Wānanga o Waitaha
CHRISTCHURCH NEW ZEALAND

ISSUE 4/06 **MAY 2006**

THIS ISSUE

1. Graduation
2. Conferences & Visits
3. Papers Submitted
4. Departmental Visitors
5. "What Is? " – an article by Douglas Bridges

GRADUATION – APRIL 2006

Congratulations and best wishes to all those who graduated last month.

Damian Campbell & Gabrielle Christenhusz head the procession to the Town Hall

Gabrielle Christenhusz BSc (Hons) & brother Gerard

CONFERENCES & VISITS

Marco Reale & **Carl Scarrott** will be attending the International Workshop in Statistical Modelling at the National University of Ireland Galway in July 2006. **John Newell** tells us that this 2006 conference marks the 100th anniversary of the publication of the first in a series of papers on small sample inference by William Gossett (aka 'Student') while working for Guinness breweries in Dublin. John will no doubt ensure that Marco and Carl honour Gossett's memory in the time-honoured tradition!

William Gossett

Chris Price: Computational Techniques & Applications Conference (CTAC 2006), Townsville, Australia 2-5 July 2006

(contin.)

CONFERENCES & VISITS (contin.)

Klaas Hartman: 2006 Winter School of Mathematical & Computational Biology, Brisbane, 26-30 June 2006, followed by the Computational Techniques & Applications Conference 2006, in Townsville, Australia 2-7 July 2006. Klaas will be giving a talk on *The Economic Optimality of Learning from Marine Reserves – a Stochastic Dynamic Programming Approach.*

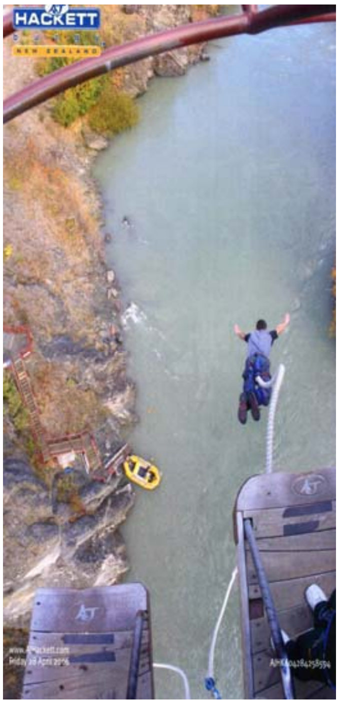
Ben Martin recently spent a week at the University of Auckland visiting Eamonn O'Brien and Laura Ciobanu. He gave a talk entitled *Lattices, Local Fields and Automorphism Groups of Trees.*

PAPERS SUBMITTED

O Gascuel & **M Steel**: *How Neighbour-Joining Minimizes Evolution* (Molecular Biology and Evolution)

R Schmid, S Shuster, M Steel & **D Huson**: *ReadSim – a Simulator for Sanger and 454 Sequencing* (Bioinformatics)

DEPARTMENTAL VISITORS



DRINKING IN THE SCENERY!

John Newell would have us believe that this is not a photo of a departmental visitor having a good time in Queenstown. Rather, it shows a dedicated Statistician demonstrating the practical applications of the "Method of Steepest Descent"!

("yeah, right!")

ALLAN WILSON CENTRE VISITORS

Bhalchandra Thatte (Rm 720), a Post-Doctoral Fellow from Massey University, is currently visiting to work on the theory of pedigree reconstruction.

Tobias Thierer (Rm 620), who completed a MSc. with us in 2004 and who is now visiting from Auckland where he is working with colleagues in bioinformatics.

B Thatte

T Thierer

Visitors	Name of Organization	From	To	Room	Extn
Dr John Holt	Massey University, Albany	15 Mar 2005	10 Apr 2007	502	7663
Ramona Schmid	Tübingen University, Germany	6 Jan 2006	6 Jun 2006	620	7431
Dr David Borchers	University of St Andrews, Scotland	23 Jan 2006	11 Aug 2006	501	8376
Dr John Newell	University of Ireland, Galway	1 Feb 2006	24 Jun 2006	501	8376
Prof. Daniel Huson (E)	Tübingen University, Germany	15 Feb 2006	30 Jun 2006	624	8877
Dr Bhalchandra Thatte	Massey University	1 Apr 2006	1 Oct 2006	616	8876
Thierer, Tobias	Auckland University	24 Apr 2006	16 Jun 2006	620	7431
Prof. Stephen Gardiner (E)	University College Dublin, Ireland	01 Jul 2006	31 Aug 2006	605	8028
Prof Christian Robert (E)	Université Paris Dauphine, France	8 Jul 2006	20 Aug 2006	607	8875
Prof. Jeremy Levesley (E)	University of Leicester, England	9 Jul 2006	14 Sep 2006	605	8028

3.

"What is ... reverse mathematics?"

By Douglas Bridges

This is a branch of mathematical logic founded by Harvey Friedman in the early 70s, in which we try to determine which axioms are needed to prove which theorems. To discuss theorem T, we work in an axiomatic framework A that is strong enough for us to express T but is ideally little, if any, stronger. In order to prove T, it would certainly suffice to add T itself to the axioms of A; but the aim of reverse mathematics is to see which of a small set of axioms, each more elementary than T, we need to add to A in order to prove T. The standard formal system used in classical reverse mathematics is second-order arithmetic plus two additional principles; known as the *base system,* this is denoted by RCA_0.

RCA_0 is strong enough to enable us to prove such things as the Baire category theorem in a complete separable metric space, the intermediate value theorem, the Banach–Steinhaus theorem for a sequence of continuous linear operators between separable Banach spaces, and even a weak version of Gödel's completeness theorem. However, to prove the Heine–Borel theorem for the closed unit real interval, the Brouwer fixed point theorem on the disc, or the Hahn–Banach theorem for a separable normed space, we need to work in WKL_0, which is RCA_0 augmented by the *weak König lemma:* every infinite subtree of the full binary tree (the tree of all finite sequences of 0's and 1's) has an infinite path. To prove the Bolzano–Weierstrasz theorem, we need to go to the next strongest system in the hierarchy based on RCA_0. Altogether, there are five axiomatic systems, including RCA_0, which turn out to be enough for the development of large parts of mathematics.

Josef Berger and I are working on a Marsden project on **constructive reverse mathematics**. In this project we use intuitionistic logic, rather than classical logic, and for the most part we work, not with formal systems, but in the informal manner of the normal mathematician (as distinct from the mathematical logician). There are two aspects of our work. First, we want to classify constructive theorems according to a few basic principles that must be added to constructive ZF set theory (CZF) in order to prove them using intuitionistic logic. Secondly, we want to identify which of a small number of fundamental nonconstructive principles (the strongest being the law of excluded middle) must be added to CZF in order to prove a given nonconstructive theorem of classical mathematics.

A good illustration of these two aspects is given by the classical theorem:

> (*) *If* $f : [0,1] \to \mathbb{R}$ *is uniformly continuous, then there exists* $x_1 \in [0,1]$ *such that* $f(x_1) \geqslant f(x)$ *for all* $x \in [0,1]$.

It is well known that with CZF and intuitionistic logic, (*) is equivalent to the nonconstructive principle

> LLPO: *For any binary sequence* $(a_n)_{n\geqslant 1}$ *with at most one term equal to* 0, *either* $a_{2n} = 0$ *for all* n *or else* $a_{2n+1} = 0$ *for all* n.

This result accords with the second aim of our project.

Now, sometimes a prior knowledge of uniqueness is the key to proving existence constructively. This suggested to us that the following form of (*) might be provable within CZF:

> (**) *If* $f : [0,1] \to \mathbb{R}$ *is uniformly continuous, and if for all distinct* x, x' *in* $[0,1]$ *either* $f(x) < \sup f$ *or* $f(x') < \sup f$*, then there exists* $x_1 \in [0,1]$ *such that* $f(x_1) \geqslant f(x)$ *for all* $x \in [0,1]$.

However, this is not the case. In the spirit of the first aim of our project, with CZF and intuitionistic logic the statement (**) turns out to be equivalent to a version of Brouwer's fan theorem (which itself is classically equivalent to König's lemma, a stronger result about binary trees than the weak König lemma mentioned earlier).

We hope that constructive reverse mathematics, which was pioneered by Hajime Ishihara around 2000, will prove as fruitful in clarifying the concepts and methods of computable mathematics as classical reverse mathematics has for classical mathematics over the last thirty years.

References

[1] J. Berger, D.S. Bridges, P.M. Schuster, 'The fan theorem and unique existence of maxima', to appear in J. Symbolic Logic.

[2] H. Friedman, 'Some Systems of Second Order Arithmetic and Their Use', Proceedings of the 1974 International Congress of Mathematicians, Vol. 1, (1975), 235-242.

[3] S.G. Simpson, *Subsystems of Second Order Arithmetic,* Perspectives in Mathematical Logic, Springer–Verlag, Berlin, 1999