Pre-Entry Test for MATH101: Solutions

Each completely correct question is worth one mark. If a question has two parts, each correct part is worth half a mark. The test is out of a total of 16 marks, and we would expect students enrolling in MATH101 to get at least 75% or 12/16 correct.

1. Solve the following problems. Answers:
   \( (a) \ 3 - 5 \times 2 = -7 \) \hspace{2cm} \( (b) \ 6 + 2 \div 4 = \frac{13}{2} \)

2. Convert the following decimals and percentages into fractions. Answers:
   \( (a) \ 0.405 = \frac{405}{1000} = \frac{81}{200} \) \hspace{2cm} \( (b) \ 0.1\% = \frac{1}{1000} \)

3. (a) What is 5% of 1,000? Answer: 50
   (b) If the original retail price of an item is $160.00, what is the price of the item when it is reduced by 25% in a sale? Answer: $120.

4. Evaluate and express as a single fraction in lowest terms (that is, give your answer in the form \( \frac{p}{q} \) where \( p \) and \( q \) have no common factor). Answers:
   \( (a) \ \frac{1}{3} + \frac{5}{6} = \frac{7}{6} \) \hspace{2cm} \( (d) \ \frac{2}{3} \div \frac{3}{4} = \frac{2}{3} \times \frac{4}{3} = \frac{8}{9} \)

5. A group of three investors buy some shares together. John puts in $2000, Steve puts in $3000, and Joanne puts in $4000. A year later they receive their first dividend of $360. How should they share out this amount?

   Answer: John receives $80, Steve receives $120, and Joanne receives $160.

6. Expand and simplify the expression \( (2x - 3y)^2 \). Answer: \( 4x^2 - 12xy + 9y^2 \).

7. Given that \( x = 1 \), \( y = 5 \), and \( z = 9 \), evaluate the following expressions.
   \( (a) \ 4(x + y)^2 \quad \text{Answer: 144.} \) \hspace{2cm} \( (b) \ -5xy + \sqrt{z} \quad \text{Answer: } -22. \)
8. Solve the equation $0.04x = 20$. Answer: $x = 500$.

9. Solve the quadratic equation $x(x - 3) = 0$. Answer: $x = 0, 3$

10. Solve this pair of simultaneous equations. Answer: $x = 5, y = 3$.

\[
\begin{align*}
    x + y &= 8 \\
    2x - y &= 7
\end{align*}
\]

11. Give the equation of the line through the points $(1, 1)$ and $(-1, 2)$. Answer: $y = -\frac{1}{2}x + \frac{3}{2}$.

12. Plot the graph of $y = (x - 1)(x + 3)$.

   **Note:** To get the mark for this question, you should have labelled the $x$- and $y$-axes and given a suitable scale on each axis.
13. Consider the following triangle.

\[
\begin{array}{c}
\text{3} \\
\downarrow \\
\text{4} \\
\end{array}
\quad \theta
\]

(a) Find the length of side \(c\). Answer: \(c = 5\).

(b) Find \(\tan \theta\). Answer: \(\tan \theta = \frac{3}{4}\).

14. Plot the graph of \(y = \cos(x)\) from \(x = 0^\circ\) to \(x = 360^\circ\).

Note that the graph should have labels on the \(x\) and \(y\) axes, a suitable scale on both axes, and have the correct amplitude and period.

15. A function is given by \(f(x) = x^2 - 3x + 6\). Find the gradient of this function at the point where \(x = -1\). Answer: \(-5\).

16. (a) Evaluate \(\int (3x^2 + 1) \, dx\). Answer: \(x^3 + x + C\)

(b) Use the result from (a) to find \(\int_0^2 (3x^2 + 1) \, dx\). Answer: 10.