Pre-Entry Test for EMTH118/MATH102 Solutions

Each completely correct question is worth one mark. If a question has two parts, each correct part is worth half a mark. The test is out of a total of 16 marks, and we would expect students enrolling in EMTH118 or MATH102 to get at least 75% or 12/16 correct.

1. If $x^2 + 8x + 5 = (x + a)^2 + b$, find the values of $a$ and $b$. Answer: $a = 4$, $b = -11$.

2. Solve $\ln(2x + 1) - \ln(x - 1) = 2 \ln 4$. Answer: $x = \frac{17}{14}$.

3. If $f(x) = x^2 + 1$, give the equation of the line that passes through $(1, f(1))$ and $(3, f(3))$. Answer: $y = 4x - 2$.

4. If $f(x) = x^3 - 2x$, find and simplify $f(x - 2)$. Answer: $f(x - 2) = x^3 - 6x^2 + 10x - 4$.

5. Solve $3 - p < p + 4$. Answer: $p > -\frac{1}{2}$.

6. Solve the following system of equations. Answer: $x = -2$ and $y = 1$.

   \[
   \begin{align*}
   2x + 3y &= -1 \\
   3x - 5y &= -11
   \end{align*}
   \]

7. Find where the line $2x - y = 1$ intersects the circle $x^2 + y^2 = 2$. Answer: $(1, 1)$ and $\left(-\frac{1}{5}, -\frac{7}{5}\right)$.
8. Sketch the graph of \( y = (x - 1)(3 - x)(x + 2) \).

Note: To get the mark for this question, you should have labelled the \( x \)- and \( y \)- axes and given a suitable scale on each axis.

![Graph of \( y = (x - 1)(3 - x)(x + 2) \).]

9. Sketch the graph of the trigonometric function \( y = \tan \theta \) for \(-\pi \leq \theta \leq \pi\).

Note that the graph should have labels on the \( x \)- and \( y \)- axes, a suitable scale on both axes, and show the vertical asymptotes at \( x = -\frac{\pi}{2} \) and \( x = \frac{\pi}{2} \).

![Graph of \( y = \tan \theta \) for \(-\pi \leq \theta \leq \pi\).]
10. Find the equation of the tangent line to the curve \( y = 3x - x^2 \) at the point \((4, -4)\).
   Answer: \( y = -5x + 16 \).

11. Differentiate the function \( y = e^x - \frac{1}{x} \).
   Answer: \( \frac{dy}{dx} = e^x + \frac{1}{x^2} \).

12. Find the derivative of the function \( f(x) = x^2 \sin x \).
   Answer: \( f'(x) = 2x \sin x + x^2 \cos x \).

13. Differentiate \( f(x) = \sqrt{x^2 - 1} \).
   Answer: \( \frac{x}{\sqrt{x^2 - 1}} \).

14. Evaluate \( \int_0^3 (t^2 + 3t - 6) \, dt \).
   Answer: \( \frac{9}{2} \).

15. Find the integral \( \int \frac{2x}{\sqrt{x^2 + 4}} \, dx \).
   Answer: \( 2\sqrt{x^2 + 4} + C \).

16. Find the area under the curve \( y = \sqrt{x} \) from \( x = 1 \) to \( x = 4 \).
   Answer: \( \frac{14}{3} \).