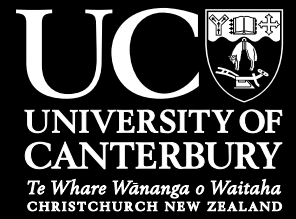


Department of Mathematics and Statistics  
College of Engineering



Honours 2010

*Courses and  
Projects*

10

# Department of Mathematics and Statistics

## Honours 2010

Welcome to Honours! Honours is an additional year in which you study selected mathematical and/or statistical topics in depth. There is a change of emphasis from the preceding undergraduate years: courses tend to be more focused on a specific problem or class of problems, rather than attempting to give a broad coverage of a branch of Mathematics or Statistics as was done in your earlier undergraduate years. Also in your Honours year, there is the possibility of a project in which you investigate some problem with the assistance of a member of staff. Depending on the nature of the problem, this may involve literature searches in the library, the use of various computing packages (for example MATLAB, Maple or R) on the Departmental computing system, resources on the internet, or even proving new theorems. At the end of the project, you produce a written report and give an oral presentation.

Who should think about Honours? If you view Mathematics and Statistics as more than a means to an end, then the Honours year will be a year well spent. The Honours year provides that additional mental edge, as well as some specific technical and (in the project) presentational skills that potential employers find valuable. If you intend to proceed on to graduate work, Masters or PhD, then Honours is a good stepping-stone.

To enter Honours in Mathematics, you will need at least 56 points from MATH310-399, plus at least 28 points from 300-level MATH, STAT or other approved courses. For Honours in Statistics, you need at least 56 points of 300-level STAT courses and another 28 points from 300-level STAT, MATH or other approved courses. Normally you should have maintained at least a B+ average in these papers. Precise details are given in the University Calendar. There is also a joint Mathematics and Statistics Honours degree (see UC enrolment handbook).

The Department offers both a BA(Hons) and a BSc(Hons). The most appropriate degree is best decided on a case-by-case basis. If you decide that you are interested in Honours, you should see the Head of Department, Assoc. Prof. Jennifer Brown, or the Honours Coordinator Dr Chris Price to discuss your options. You should do this before the enrolment week next year. (In fact, there is no reason not to do it now!).

Courses are packaged in 20-lecture modules that will run at two lectures a week for a semester. Any proposed programme of study for Honours requires the approval of the Head of Department. It is highly unlikely that any proposed programme that has a high workload in one semester will be approved, so you should try to construct a programme that balances your workload evenly over both semesters.

## **1. BSc(Hons)**

In the Science faculty, Honours may be completed in Mathematics, in Statistics, in Mathematics and Statistics, in Mathematical Physics, in Computational and Applied Mathematics (CAMS), in Mathematics and Philosophy or in Economics and Mathematics. Students enrolled for a BSc(Hons) in Mathematics and/or Statistics are required to complete eight 400-level courses, as well as a project which is worth the equivalent of two courses. In the Mathematical Physics programme, courses from both Physics and Mathematics are required. Students who are interested in this programme should contact Dr Ben Martin or Dr David Wiltshire (Physics). If you are interested in the CAMS programme, see Prof. David Wall or Assoc. Prof. Rick Beatson. If you are interested in the Mathematics & Philosophy programme, see Prof. Douglas Bridges or Dr Philip Catton (Philosophy). For the Economics and Mathematics programme, see Prof. Douglas Bridges or Dr Seamus Hogan (Economics).

## **2. BA(Hons)**

In the Arts faculty, Honours may be completed in Mathematics or in Statistics. A BA(Hons) has the same requirements as a BSc(Hons).

## **3. PG Dip Sci**

A Post Graduate Diploma in Science (PGDipSci) can be obtained by completing eight 400-level papers in one year. These courses normally do not include the honours project (MATH/STAT449). Entry requirements

are as for a BSc(Hons), except that you are not required to have a B+ average. It is very strongly recommended that your average grade in your majoring subject at stage 3 is at least a C+. A PG Dip Sci may be used as the first part of an MSc.

## **4. Proposed Courses for 2010**

The proposed courses for 2010 are outlined in this booklet. The final decision about which courses are to be offered will depend on the availability of staff and on student interest. The Department reserves the right to cancel any course that does not attract four or more students. It is also possible (and in some cases desirable) to include in your Honours programme appropriate courses from other departments or 300-level courses from Mathematics or Statistics. In the latter case, extra work is usually required to bring the paper to a level consistent with other 400-level courses. Note that, subject to having the required background, any STAT courses may be included in a Mathematics degree and vice versa.

## **5. Proposed Projects for 2010**

A broad range of possible projects is outlined in this booklet. However, this list is not exhaustive and other possibilities for projects are certainly possible. Project supervision is by mutual agreement of the supervisor and student. You should arrange your project by the end of the first week of term in 2010. It is suggested that you seek out possible supervisors during the enrolment week (or before if you have a clear idea of what sort of project

interests you). You will hand in a written report on the project in late September; part of your project assessment will be an oral presentation in Term 4.

## **6. Department Scholarships**

The Department offers up to three full fees scholarships at domestic level. These only cover fees for Mathematics and Statistics courses. The Department also offers up to eight \$1,000 scholarships, which are restricted to those doing BSc(Hons), BA(Hons) or PGDipSc in the Department of Mathematics and Statistics. For further details, see the Department Handbook.

## **7. Important Meeting**

During the first week of Term 1, 2010, there will be a meeting for all students enrolling in at least one 400-level paper in Mathematics and/or Statistics. It is important that all prospective Honours students are at the meeting. As well as providing general information, the meeting determines which 400-level courses will run in the first and second semesters. (Most first semester Honours courses will start in the second week of Term 1.)

## **8. Engineering Mathematics 600 level**

A number of the honours papers are permanently double coded as 600 level (i.e. Masters) engineering mathematics papers. Further honours papers may be given EMTH600 special topic codes to meet student needs as they arise. For more information you should contact the Honours Coordinator, Dr Chris Price.

# 400-level courses

The semesters indicated for these courses are listed as a guide only. The courses may be offered in semester 1 or semester 2 depending on the availability of the lecturer. All 400-level courses are offered subject to sufficient student demand and staff availability, which will be determined at the beginning of each semester. For an up-to-date view of the department's current offerings, please see the department's web page, which is located off the main University web page.

## MATHEMATICS

### **MATH401** **Dynamical Systems 1**                      **15 points**

MATH401-10S1 (C)

Dynamical systems is a rapidly developing branch of Mathematics with growing applications in diverse fields from traditional areas of applied mathematics to numerical analysis, biological systems, economic models and medicine.

It is often difficult or impossible to write down an exact solution to systems of nonlinear equations. The emphasis in this course will be on qualitative techniques for classifying the behaviour of nonlinear systems, without necessarily solving them exactly. Both main types of dynamical system will be studied: discrete systems, consisting of an iterated map; and continuous systems, consisting of nonlinear differential equations. Topics covered will include: chaotic behaviour of simple 1D maps; period-doubling bifurcations; phase portrait analysis; methods for determining stability of fixed points and limit cycles; centre manifolds; and symbolic dynamics. MATH401 and MATH402 are excellent companion courses for students seeking broad knowledge of dynamical systems. MATH401 and MATH402 may be taken independently of one another.

**Enquiries:** Mike Plank

### **MATH402**

### **Dynamical Systems A**                      **15 points**

MATH402-10S1 (C)

This course will cover two mathematical topics from dynamical systems, using the notation of entropy as an organizing principle: (i) topological dynamics, with an emphasis on chaos in one-dimensional maps and subshifts of finite type; (ii) statistical properties of dynamical systems, including measure theoretic entropy; (iii) applications of dynamical systems to mechanics, in particular, the dynamics of conservative systems, and some of the central results of dynamical systems in Lagrangian mechanics and Hamiltonian mechanics; and (iv) applications of dynamical systems and bifurcation theory to biological systems. MATH401 and MATH402 are excellent companion courses for students seeking broad knowledge of dynamical systems. MATH401 and MATH402 may be taken independently of one another. Math402 does not require MATH401 as a pre-requisite.

**Enquiries:** Rua Murray, David Wall

### **MATH405**

### **Bioinformatics**                                      **15 points**

MATH405-10xx (C)

Bioinformatics is currently a fast-growing field of research. This course will address one general question in this area, namely, what can genetic sequences tell us about the evolution of species? Topics covered will include phylogenetic trees and networks, distance and character based approaches to tree reconstruction, Markov models of sequence evolution and population genetics. The course will mainly use discrete mathematical techniques (particularly algorithms, graph theory and probability theory) and will be mostly self-contained, and suitable for a mathematically mature student from other related disciplines (e.g. computer science, biological sciences, physics). The semester in which this course

is offered will be advised at a later date on the department's web page.

**Enquiries:** Mike Steel

## **MATH406**

### **Mathematical Models in Biology** **15 points**

MATH406-10S2 (C)

How did the leopard get its spots? Why should children be vaccinated against measles? This course will try to answer these (and other) questions by using mathematical models to examine biological phenomena.

Some knowledge of dynamical systems and a familiarity with Maple (or MATLAB) are useful pre-requisites to this course.

**Enquiries:** Alex James

## **MATH407**

### **Population Genetics** **15 points**

MATH407-10S1 (C)

Population genetics models the evolution of genes in populations. These models can run either forward in time (given a population, what will the distribution of genes and alleles be in future generations?), or backwards in time (given a sample DNA from the present, what process describes the ancestry of the sample?). This course will examine both approaches, with particular attention given to coalescent theory, which uses a stochastic process to model events backwards in time.

Particular applications studied in the course can depend on the interests of the students, but may include simulation, recombination, migration, diffusion theory (genetic drift), estimation of population parameters, disease association mapping, and phylogenetics.

**Enquiries:** James Degnan

## **MATH409**

### **Cryptography 2** **15 points**

MATH409-10S2 (C)

Cryptography is the science of making and breaking secret codes: encryption is what keeps our credit card details safe when we send them over the internet. In this course we will study the mathematics behind some of the main encryption systems in current use. These systems draw on ideas from algebra, geometry, number theory and probability theory. The course is aimed at students majoring in Computer Science or Mathematics. See MATH324.

**Enquiries:** Ben Martin

## **MATH410**

### **Approximation Theory** **15 points**

MATH410-10S2 (C)

The first part of this course will concentrate on the fundamentals of approximation of functions of one variable. Central topics will be approximation by algebraic and trigonometric polynomials, and the existence, characterisation and uniqueness of best approximations from finite dimensional normed linear spaces.

In the latter part of the course we will develop some more recent topics such as smoothest interpolation of functions of several variables and computational geometry in 3D.

Approximation Theory lies at the interface of many specialties. As such its study involves an interesting mix of pure and applied mathematics, numerical analysis and advanced computation.

Recent applications of Approximation Theory here at Canterbury include fitting surfaces to noisy point clouds applied, for example, to the manufacture of artificial limbs, and, fitting geophysical data sets such as grade measurements from drill holes in mines.

**Enquiries:** Rick Beatson

## **MATH412**

### **Unconstrained Optimization** **15 points**

MATH412-10S2 (C)

This course looks at the minimization of smooth functions of several variables. The first part of the course examines gradient based methods using line searches, including Newton, quasi-Newton, and conjugate gradient methods. A selection of other topics are then introduced, including trust region methods and methods for constrained optimization. Demonstration software is used to illustrate aspects of various algorithms in practice.

**Enquiries:** Chris Price

## **MATH414**

### **Computational Methods** **15 points**

MATH414-10S2 (C)

Iterative Krylov methods for large linear systems.

- Based around book by Henk Van der Vorst (of that title).
- Survey of iterative methods.
- Conjugate gradient, GMRES, MINRES methods: derivation, algorithms, convergence, preconditioning.
- Semi-reading course based around the book, and Watkins MATH353 book.
- MATLAB programming involved.

**Enquiries:** Bob Broughton

## **MATH416**

### **Differential Systems** **15 points**

MATH416-10S1 (C)

Moving frames and exterior differential systems form a natural backdrop for the study of problems in geometry and partial differential equations. In particular, most (if not all) physical systems exhibit symmetry and so embody geometric content in the differential equations that describe the

physics. These systems are naturally described in the coordinate free approach of moving frames. Integrability conditions, the "size" of the solution space and the existence of "singular" branches of the solution space for systems of partial differential equations can be readily found in the moving frame approach.

This course will introduce moving frames and exterior differential systems with an emphasis on the conceptual and operational issues. The "standard" vector calculus will be revisited with the aid of differential forms. Their application to simple geometric problems and the reformulation of Maxwell's equations as an exterior differential system will be considered. If time permits (and depending on student interest) either the exterior differential system formulation of Einstein's field equations or application of moving frames to computer recognition of objects will be considered.

Prospective students should have familiarity with partial differential equations and vector calculus.

**Enquiries:** Mark Hickman

## **MATH420**

### **Hilbert Spaces** **15 points**

MATH420-10S1 (C)

The theory of Hilbert spaces is fundamental in many areas of modern mathematical analysis, having a clear and easy-to-grasp geometric structure, just like Euclidean spaces. However, unlike Euclidean spaces, Hilbert spaces may be infinite dimensional. The course will be self-contained, introducing important spaces (especially  $L^2(m)$ ), operators on them, and basic spectral theory. Applications in dynamical systems (von Neumann's ergodic theorem) and quantum mechanics will be included as time permits. Prior exposure to MATH343 would be an asset, but is not mandatory.

**Enquiries:** Rua Murray

## **MATH424**

### **Wavelets and Data Compression 15 points**

MATH424-10S1 (C)

What are wavelets? Wavelets and orthogonal decompositions. Wavelet transforms. Applications to signal analysis and data compressions. The point of view will be very much an applied one.

Recommended Preparation: Linear algebra (at about 200 level) and an appreciation for Fourier series and transforms.

**Enquiries:** Peter Renaud

## **MATH425**

### **Fourier Transforms and Distribution Theory 15 points**

MATH425-10S2 (C)

Operational calculus (Distribution theory) had been used by engineers (Heaviside) and physicists (Dirac) long before it was developed by L. Schwartz into a rigorous coherent theory. Before Schwartz it was already used successfully by S. L. Sobolev in the study of partial differential equations. Since the early 1950's distribution theory has become an indispensable tool in many branches of mathematical analysis.

This course is an introduction to Fourier analysis and distribution theory. Amongst the topics covered are: The Fourier Transforms, Poisson summation formula, Sampling theorems, Fourier transform of distributions, Paley-Wiener theorem, the continuous wavelet transform.

Fourier analysis is a fertile meeting ground of real, complex, and functional analysis. In this course this aspect will be highlighted. To make the course self contained, background material from other fields will be given (and explained).

**Enquiries:** Qui Bui

## **MATH427**

### **Lie Groups and Lie Algebras 15 points**

MATH427-10S1 (C)

Lie groups are an essential tool in many areas of Mathematics and Physics. They are groups that also carry an analytical structure and are often found as groups of symmetries of 'nice' mathematical objects like geometries or differential equations. The most important Lie groups are finite-dimensional and occur as groups of matrices over real or complex numbers, like the group of all rotations of Euclidean 3-space.

Every Lie group has an associated Lie algebra which is a very good linear approximation of the group. Many properties of the Lie group can be deduced from its algebra.

The course gives an introduction into the theory of finite-dimensional Lie groups, and their associated Lie algebras and linear representations.

**Enquiries:** Günter Steinke

## **MATH428**

### **Topology 15 points**

MATH428-10S2 (C)

Topology, colloquially known as 'rubber-sheet geometry', is the study of continuity in an abstract setting. Topological notions underpin, or are used in, many areas of mathematics, ranging from analysis to algebraic geometry and even set-theory. Accordingly, the fundamentals of point-set topology are an essential part of the training and armoury of the modern research mathematician. In this course, we introduce topological spaces and study continuity, limits, and many other important notions, in that setting. We then take first steps in algebraic topology, with its wonderful interplay between pure algebra and pure topology.

The topics will be drawn from the following: Topological spaces, continuous functions; filters and convergence; compact and

connected spaces; separation properties. An introduction to homotopy theory and to degree theory, with applications (for example, the Brouwer fixed-point theorem, and the fundamental theorem of algebra).

**Enquiries:** Douglas Bridges

## **MATH429**

### **Combinatorics 15 points**

MATH429-10S2 (C)

Matroids (also called combinatorial geometries) are precisely the structures that underlie the solution of many combinatorial optimization problems. These problems include scheduling and timetabling, and finding the minimum cost of a communications network between cities. Given this, it is perhaps surprising that matroid theory unifies the notions of linear independence in linear algebra and forests in graph theory as well as the notions of duality for graphs and codes.

This self-contained course is an introduction to matroid theory, a branch of discrete mathematics that has basic connections with graphs, codes, projective geometries, and combinatorial optimisation. The course is intended for students majoring in Mathematics or Computer Science.

**Enquiries:** Charles Semple

## **MATH432**

### **Foundations of Mathematics 15 points**

MATH432-10S2 (C)

An introduction to the philosophy of mathematics, classical and intuitionistic logic, set theory and Godel's theorems.

See MATH 336.

**Enquiries:** Douglas Bridges

## **MATH433**

### **Mathematics in Perspective 15 points**

MATH433-10S1 (C)

What is Mathematics? What are some of the key moments in the history of Mathematics?

What kinds of mathematical result are considered important, and why?

This course is about the history, philosophy, people and major results of Mathematics over the centuries. Since we will minimise the attention paid to technical details, the course should be accessible not only to those with a 200 level Mathematics background, but also to intellectually mature students in Philosophy and related subjects. In particular, it is strongly recommended for anyone who intends teaching Mathematics at any level from primary school onwards. See MATH301.

**Enquiries:** Clemency Montelle

## **MATH437**

### **Representation Theory 15 points**

MATH437-10S1 (C)

The set  $GL(n, \mathbb{C})$  of invertible complex  $n \times n$  matrices forms a group under multiplication. Given  $n$ , a representation of a group  $G$  is a homomorphism (structure-preserving map) from  $G$  to  $GL(n, \mathbb{C})$ . The idea of representation theory is to study abstract groups by studying their representations. Representation theory has applications in theoretical physics: whenever there is symmetry in a physical problem, there is usually a representation of the associated symmetry group. This course will give an introduction to representation theory, first for finite groups, then for compact Lie groups (such as the special orthogonal group  $SO(3)$  of rotations in 3-space).

Preparation: Some familiarity with groups will be assumed (the definition of a group, subgroups, homomorphisms, quotient groups, etc.). If your background in group theory is weak, you will probably need to do some preparatory work at the beginning to catch up. Notes summarising the necessary background material will be available from the course website before the semester begins.

**Enquiries:** Ben Martin

## **MATH438**

### **Advanced Scientific computing ---**

#### **PDEs 15 points**

MATH438-10xx (C)

This course will take an advanced look at the numerical solution of partial differential equations. It will focus on (i) variational methods of solving hyperbolic, parabolic, and elliptic equations; and (ii) methods of solving PDEs having conservation properties. Convergence theory of these methods will be covered. The semester in which this course is offered will be advised at a later date on the department's web page.

**Enquiries:** David Wall

## **MATH439**

### **Topics in Algebra 15 points**

MATH439-10S1 (C)

Groups most often arise as collections of symmetries of mathematical systems like patterns in the plane or space, graphs or geometries. From its origins in the 18<sup>th</sup> century when the need arose to formalize the notion of symmetries of algebraic equations, group theory has developed into a flourishing and exciting branch of mathematics. It has remained extremely useful with, for example, applications to enumeration, geometry, graph theory, physics, and chemistry.

The course gives a thorough introduction to groups, their internal structure and applications. See MATH322.

**Enquiries:** Günter Steinke

## **MATH442**

### **Special Topic in Mathematics (Viscous Fluid Dynamics) 15 points**

MATH442-10S2 (C)

This course is intended as a continuation of Inviscid Fluid Dynamics although is not set as a prerequisite. In the course we will give a quick review of the conservation equations

and derive constitutive relations for Newtonian and generalized Newtonian fluids. Afterwards we will concentrate on examples of flows at low Reynolds numbers, for example highly viscous flows, porous media flows, thin films and lubrication theory to name some.

**Enquiries:** Miguel Moyers-Gonzalez

## **MATH443**

### **Metric, Normed and Hilbert Spaces 15 points**

MATH443-10S1 (C)

This course introduces those parts of modern analysis that are essential for many aspects of Pure and Applied Mathematics, Physics, Economics, Finance, and so on. For example, if you want to understand the convergence of numerical algorithms, approximation theory, quantum mechanics, or the economic theory of competitive equilibrium, then you will need to know something about metric, normed and Hilbert spaces. See MATH343.

**Enquiries:** Chris Price

## **MATH444**

### **Special Topic in Mathematics (Inviscid fluid dynamics) 15 points**

MATH444-10S1 (C)

Fluid Dynamics is the study of how aeroplanes fly, flags flap, blood flows, rivers run, waves break, and galaxies turn. Anything remotely fluid-like, from desert sand dunes through water fountains to the slow flow of mountains, can be described and studied by the methods of fluid mechanics. In this course, we will learn the fundamental definitions, theorems, and results of the basis of the subject, concentrating on flows which can reasonably be approximated by modelling a non-sticky (or *inviscid*) fluid. This course will be excellent preparation for the Semester 2 course in viscous fluid dynamics.

**Enquiries:** Phil Wilson

## **MATH449**

### **Project 30 points**

MATH449-10W (C)

A whole year research project in mathematics (see Honours projects).

**Enquiries:** Department of Mathematics and Statistics Reception

## **MATH491**

### **Summer Research Project 15 points**

MATH491-09SU2 (C)

This 150 hour course provides students with an opportunity to develop mathematical research skills and to extend and strengthen their understanding of an area of mathematics.

**Enquiries:** Alex James, Ben Martin

## **STATISTICS**

## **STAT405**

### **Bioinformatics 15 points**

STAT405-10xx (C)

Bioinformatics is currently a fast-growing field of research. This course will address one general question in this area, namely, what can genetic sequences tell us about the evolution of species? Topics covered will include phylogenetic trees and networks, distance and character based approaches to tree reconstruction, Markov models of sequence evolution and population genetics. The course will mainly use discrete mathematical techniques (particularly algorithms, graph theory and probability theory) and will be mostly self-contained, and suitable for a mathematically mature student from other related disciplines (e.g. Computer Science, Biological Sciences, Physics). The semester this

course is offered in will be advised at a later date on the department's web page.

**Enquiries:** Mike Steel

## **STAT446**

### **Generalised Linear Models 15 points**

STAT446-10S1 (C)

How do you analyse data that does not fit the standard methods such as ANOVA and regression? How do you deal with data that are very non-normal, are counts rather than measurements, are correlated and have interdependencies? In this course we introduce you to the very useful toolbox of Generalised Linear Models (GLMs). This is a natural progression from understanding ANOVA, regression and multivariate techniques. We will learn about the general framework for GLMs, and how to use GLMs for analysing data. We will introduce you to the package R, and will use this software throughout the course. Some background in statistical analysis methods is necessary, and useful courses to have completed are STAT220, STAT212 and STAT315. No experience in R is necessary. See STAT319.

**Enquiries:** Jennifer Brown

## **STAT449**

### **Project 30 points**

STAT449-10W (C)

A whole year research project in Statistics (see Honours projects).

**Enquiries:** Department of Mathematics and Statistics Reception

**STAT452****Applied Statistics** **15 points**

STAT452-10S1 (C)

STAT452-10S2 (C)

A project-based course in Applied Statistics. Project choice is by mutual arrangement between the student and the project supervisor. Students should make sure supervision arrangements are in place before enrolling in the course.

**Enquiries:** Any Statistics lecturer.

**STAT455****Sampling Methods** **15 points**

STAT455-10S1 (C)

This course looks at practical methods for gathering new data, the raw material of Statistics. See STAT312.

**Enquiries:** Jennifer Brown

**STAT456****Time Series and Stochastic Processes** **15 points**

STAT456-10S1 (C)

Here we explain some techniques to model observations taken sequentially over time. This kind of data is very common in Biology, Environmental Sciences, Economics and Finance. Time series methods are widely used for forecasting. This course is application oriented, and computers are used to analyse real time series data. See STAT317.

**Enquiries:** Marco Reale

**STAT458****Applied Stochastic Modelling** **15 points**

STAT458-10S2 (C)

This course studies Markov processes. These are sequences of random variables where dependence on past history may be reduced to dependence only on the most

recent value. Such a simple form of dependence leads to a remarkably rich variety of models.

**Enquiries:** Subject to lecturer availability.

**STAT460****Extreme Value Statistics** **15 points**

STAT460-10S1 (C)

This course aims to develop the theory and methods for modelling the extremes of random processes. Extreme value theory moves away from more traditional statistical techniques where the aims are to model the usual (or in some sense average) behaviour, to consider the unusual or rare events. It has received wide application in many fields where the risk associated with rare events are of concern, e.g. finance/economics, hydrological modelling, climate change, engineering (structural design) and material science (material fatigue/failure).

The course will cover the mathematics underlying extreme value models, statistical inference using likelihood and applications to real data, with implementation in the software package R. Recommended preparation includes second year Statistics (preferably STAT214) and at least full first year Mathematics (MATH109 or equivalent).

**Enquiries:** Carl Scarrott

**STAT462****Data Mining** **15 points**

STAT462-10S2 (C)

Data mining refers to a collection of tools to discover patterns and relationships in data, especially for large data bases. It involves several fields including data base management, statistics, artificial intelligence, and machine learning, and it has had a considerable impact in business, industry and science.

This course provides an introduction to the principal methods in data mining: data preparation and warehousing, supervised learning (tree classifiers, neural networks),

unsupervised learning (clustering methods), association rules, and the dealing with high-dimensional data (PCA, ICA, multidimensional scaling). Students will see applications from various fields, such as commerce (fraud detection, product placement, targeted marketing, assessing credit risk) and medicine (diagnostics). We will use data mining software to illustrate methods with data sets from these fields.

Students must (i) do problems that are assigned throughout the term and (ii) research an area and write an account of it; the instructor will give suggestions for topics in class. See STAT318.

**Enquiries:** Marco Reale

### **STAT463**

#### **Multivariate Statistical Methods 15 points**

STAT463-10S2 (C)

Multivariate statistical methods extract information from datasets which consist of variables measured on a number of experimental units. The application of these methods is blooming with the availability of large datasets from a wide range of scientific fields, combined with the advent of computing power to implement them. Examples abound in fields as diverse as bioinformatics, internet traffic analysis, clinical trials, finance and marketing. This course will cover the theory and applications of various multivariate statistical methods. See STAT315.

**Enquiries:** Carl Scarrott

### **STAT464**

#### **Statistical Inference 15 points**

STAT464-10S1 (C)

STAT464-10S2 (C)

This course looks at the foundations of statistical inference. It covers estimation and test of hypotheses from the classical viewpoint and includes the introduction to the Bayesian approach.

**Enquiries:** Department of Mathematics

and Statistics Reception. Subject to lecturer availability.

### **STAT470**

#### **Special Topic in Statistics (Population Genetics) 15 points**

STAT470-10S1 (C)

Population genetics models the evolution of genes in populations. These models can run either forward in time (given a population, what will the distribution of genes and alleles be in future generations?), or backwards in time (given a sample DNA from the present, what process describes the ancestry of the sample?). This course will examine both approaches, with particular attention given to coalescent theory, which uses a stochastic process to model events backwards in time.

Particular applications studied in the course can depend on the interests of the students, but may include simulation, recombination, migration, diffusion theory (genetic drift), estimation of population parameters, disease association mapping, and phylogenetics.

**Enquiries:** James Degnan

### **STAT471**

#### **Auto-validating Methods 15 points**

STAT471-10S2 (C)

The aim of the course is to identify the inherent limitations of floating-point computations, and show how to overcome these by the use of a different class of algorithms. We will describe the basics of auto-validating methods, and systematically cover various problems that can successfully be handled using C++ class libraries. These methods were used in the proofs of the existence of the Lorentz attractor and Kepler's sphere packing problem, for instance.

**Enquiries:** Raazesh Sainudiin

## **STAT472**

### **Special topic in Statistics**

**(Forecasting) 15 points**

STAT472-10S2 (C)

Forecasting plays an important role in decision making in industry, financial services, government, environmental management, public health, and many other areas. For example in an uncertain environment, the success of a business or economic policy depends on the ability of managers to foresee and prepare for the future.

This course is ideal if you are keen to advance your career as a modeller or forecaster or are working in a related area that would benefit from training in modelling or forecasting methods. It provides extensive training in forecasting and modelling techniques such as smoothing, dynamic regressions, multivariate autoregressions, state space models, and neural networks with a wide range of applications.

**Enquiries:** Marco Reale

## **STAT474**

### **Special Topic in Statistics**

**(Financial Time series) 15 points**

STAT474-10S1 (C)

This course is concerned with modelling financial volatility, and provides an econometric analysis of alternative models and techniques for analysing high frequency and ultra high frequency financial data. In each of the three primary areas of volatility modelling, namely Conditional Volatility or Generalized Autoregressive Conditional Heteroskedasticity (GARCH), Stochastic Volatility (SV) and Realized Volatility (RV), univariate volatility models of individual financial assets and multivariate volatility models of portfolios of assets, will be examined critically, the mathematical structural properties of the models will be established, the associated estimation algorithms will be developed, the statistical properties of the estimators will be derived,

and the forecasting performance will be evaluated.

Restrictions: FINC313-10S1, FINC652-10S1

**Enquiries:** Department of Mathematics and Statistics Reception

## **STAT475**

### **Independent course of study**

**15 points**

STAT475-10S1 (C)

STAT475-10S2 (C)

This course allows a student to perform directed reading of a particular topic under a Statistics lecturer. The topic choice is by mutual arrangement with the lecturer. Students should ensure these arrangements are in place before enrolling in the course. In particular Raazesh Sainudiin is offering the following topic in the first semester.

Advanced Auto-validating methods. This is a continuation of STAT471. Students who have not taken STAT471 may take the course with some additional preparation. Topics covered will include complex interval arithmetic, multidimensional global optimization, multidimensional solvers, contractors, parameter estimation, interval constraint propagation, interval-extended gradient and Hessian differentiation arithmetic, rigorous ODE and PDE solvers and randomized enclosure algorithms.

**Enquiries:** Raazesh Sainudiin

## **STAT491**

### **Summer Research Project 15 points**

STAT491-09SU2 (C)

This 150 hour course provides students with an opportunity to develop statistical research skills and to extend and strengthen their understanding of an area of statistics.

**Enquiries:** Alex James, Ben Martin

# Honours Projects

A broad range of possible projects are outlined. However, this list is not exhaustive and other possibilities for projects are certainly possible.

Project supervision is by mutual agreement of the supervisor and student.

You should arrange your project by the end of the first week of term in 2010. It is suggested that you seek out possible supervisors during the enrolment week (or before if you have a clear idea of what sort of project interests you).

You will hand in a written report on the project in late September; part of your project assessment will be an oral presentation during Term 4.

## Projects in Mathematics

### Approximation of noisy data

Rick Beatson

Many real world situations require the fitting of noisy data  $(x_i, y_i)$  with  $i=1\dots N$ . Among the many possible approaches are penalized radial basis functions where one fits a function  $s(x)$  so as to minimize a sum like

$$\frac{1}{N} \sum_{i=1}^N (y_i - s(x_i))^2 + \mu E(s)$$

This approach trades fidelity to the data given by the mean square deviation off against smoothness measured by the "energy" of  $s(x)$ . Sparsity can also be encouraged by including a 1-norm of the coefficients. There are many interesting theoretical and practical aspects of this problem. For example under what assumptions (on the space of approximations) does the solution take the standard spline smoothing form, which makes the nonlinear minimization problem linear? Also, what numerical algorithms

make the computations fast enough to be practical for large datasets?

Questions to be emphasized will be chosen to suit the background and interests of the person taking the project.

### Aspects of pipe flow

Phil Wilson

Oil, sewage, blood, bile: the flow of fluids through pipes is ubiquitous in industry and biology. Some of these flows can be reasonably approximated by assuming that the fluid is inviscid. We will consider aspects of modelling such flows in smooth curved pipes. Some knowledge of asymptotic analysis is preferable but not required. The keen and technically advanced student may progress to modelling an unsolved problem in the merger of turbulent boundary layers in curved pipes.

### Elliptic curves and cryptography

Ben Martin

Please see Dr Martin for details.

### Escape from a dynamical system

Rua Murray

Many dynamical systems have holes! The system may be born with these (in the case of a system of hard scatterers, or a billiard table --- so called "open systems"), or holes may be introduced as leaks from a closed dynamical system. Typical orbits escape from open systems with a characteristic exponential rate, depending on delicate nonlinear phenomena. The details of this project are flexible, but will include some exciting mathematics!

### Fractional calculus and the Klavier of Matheron

Rick Beatson

Fractional calculus is a beautiful part of analysis with many applications. This project will survey some of the basic theory of fractional calculus and then consider

some applications to radially symmetric functions in  $\mathbb{R}^d$ . In particular the project will consider a circle of results due to Matheron which allow one to calculate the convolution of two radially symmetric functions in  $\mathbb{R}^d$  by means of a convolution two dimensions below in  $\mathbb{R}^{d-2}$ . Matheron's results have found much practical application in geostatistics and approximation theory. He is known particularly for the turning bands method for conditional simulation in several dimensions.

## Group based cryptography

Ben Martin

Many modern cryptographic systems are based on ideas from number theory such as integer factorization. This project involves cryptographic systems based on group theory. Possible topics include braid group cryptography and Diffie-Hellman key exchange for nonabelian groups.

## History of Mathematics

John Hannah

The second half of the 19th century and the first half of the 20th century saw the introduction and development of several quite abstract branches of mathematics. Examples include abstract group theory, topology, abstract algebra (fields, rings, ideals, modules) and category theory. In this project you will follow this development in a topic of your own choosing, and in the process you will both learn some new mathematics, and try to discern the influences (whether mathematical, philosophical or cultural in origin) which led to these developments.

## How do microorganisms swim?

Miguel Moyers-Gonzalez

Organisms of various kinds ranging from the very small (for example, amoeba or spermatozoa) to the very large (for example fish, dolphin, whale), make their way through a fluid (often water) using one or more of a multitude of strategies, usually involving self-deformations. One question is

how a deformation translates into motion, another is how much energy does the resulting motion require the organism to expend. These questions have been of interest to many people over the years and the purpose of the project is to review some of the models that have been created in order to study the problem. The references provide a starting point but there are many others to be found.

## How to keep a secret

Günter Steinke

In order to open a bank vault (or to launch a missile) a code is used. However, it is not desirable to entrust the code to a single person. Instead one shares out parts of the code or information that leads to parts of the code so that only when a sufficient number of entrusted persons come together and pool their shares the code will be revealed.

The project looks at the mathematics of so-called secret sharing schemes and how they can be devised and implemented.

## Inverting the divergence operator

Mark Hickman

Algebraic computing packages such as Maple and Mathematica are adept at computing the integral of an explicit expression in closed form (where possible). Neither program has any trouble in, for example, integrating  $\log(x)/x$  to obtain  $\log(\log(x))$ . Maple, from release 9 onwards, has a limited facility to handle expressions such as

$$\frac{uv_x - vu_x}{(u-v)^2} = \frac{d}{dx} \frac{v}{u-v}$$

where  $u$  and  $v$  are understood to be functions of  $x$ . However neither program can compute the "anti-derivative" of exact expressions in more than one independent variable. For example there are no inbuilt commands that would compute

$$\begin{aligned} & u_x v_y - u_{xx} v_y - u_y v_x + u_{xy} v_x \\ &= \frac{\partial}{\partial x} (u v_y - u_x v_y) + \frac{\partial}{\partial y} (u_x v_x - u v_x) \end{aligned}$$

The aim of this project is to write a Maple library to compute (where it exists)  $F$ , where the divergence of  $F$  equals a given  $f$ .

## **Modelling problems in biology and ecology**

Alex James

These cover a wide range of topics from epidemiology through invasive species to fluid flow in bugs. Modelling techniques are primarily taken from dynamical systems, PDEs and stochastic processes. For more information please get in touch.

## **Networks of Swine Flu**

Mike Plank

Swine flu (or to use the proper name pandemic H1N1 influenza) was big news in 2009. Mathematics has a major role to play in prediction via the decisions of public health managers. The traditional tool for this is the SIR (Susceptible-Infected-Removed) model, which consists of a simple system of differential equations. Whilst models of this type have been successful on a broad scale, they usually assume that the entire population is homogeneously mixed. In reality, there is a complex web of potential disease transmission routes, and a good way to represent this mathematically is a network model. This project will investigate the effects of network structure on epidemic dynamics, and compare these to the standard SIR model.

## **Quantum computing**

Ben Martin

A quantum computer is a computer that takes advantage of ideas from quantum mechanics: rather than being in one definite state at any given time, it is in several different states, so it can do several different calculations simultaneously. In this project you will look at basic properties of quantum computers and maybe also applications such as Shor's integer factorization algorithm.

## **Tutte's 5-Flow conjecture**

Charles Semple

Network flow problems are an important class of problems in combinatorial optimization and represent a large variety of real-world occurrences.

A particular type of network problem gives rise to Tutte's 5-flow conjecture, the most outstanding conjecture in graph theory today. The purpose of this project is to investigate the progress that has been made on this problem and its connections to other areas of combinatorics. To whet your appetite, the Four Colour Theorem says that every planar graph without isthmuses has a 4-flow. While some prior knowledge of graph theory would be helpful, it is not a prerequisite for the project.

## **Sampling Theorem**

Qui Bui

The Shannon-Whittaker sampling theorem shows how to reconstruct a band-limited signal (a function with compactly supported Fourier transform) from its sampled values at a discrete set of points, using the basis function  $\text{sinc}(x) = \sin(x)/x$ . In this project the student will learn recent developments related to this theorem. The project is suitable for a student who has a reasonable background in Fourier series and the Fourier transforms.

## **Teacups, doughnuts, and pretzels**

Günter Steinke

Teacups and doughnuts are easily distinguishable: just one bite into one or pour tea onto it. Mathematically (or more precisely topologically) it is a different matter. If both were made of rubber and were ideally stretchable we could deform one into the other without ripping the rubber apart. One can, however, distinguish doughnuts and pretzels: there is no way to deform one into the other.

Teacups, doughnuts, and pretzels can be considered as rough realizations of (topological 2 dimensional) manifolds and as such can be assigned an integer, its genus.

Different genres imply distinguishable manifolds.

The project requires no baking or catering experience nor is it anticipated that you gain any. Instead it explores concepts from topology, and, in particular, the notion of (topological 2-dimensional) manifolds and their representations and characterizations.

### **Which walk is best?**

Mike Plank

You are a monkey in a forest. You are hungry. You know there is food out there, but you can't see any. What should you do? Start looking! But exactly how should you search? A random walk is one way to start, but you can usually do much better if some systematic strategy is added, rather than purely random movements. Random walks come in all shapes and sizes, so the question is: which walk is best? Some knowledge of basic probability distributions and experience of computer programming (e.g. Maple/MATLAB) would be very useful for this project.

## **Projects in Statistics**

### **Effects of skewness and non-Normality on the t-test**

Marco Reale

The t-test is the common significance test to assess the relevance of the behaviour of one variable on another one in a regression. Normality is a requirement for its exact sampling properties, however empirical evidence shows that this test is quite robust to non-Normality in the presence of symmetry, while it may be quite sensitive to skewness. This research project would examine how critical the skewness is and the effect of other distributional aspects such as bimodality.

### **Estimating gene tree proportions in the presence of unresolved maximum likelihood estimates**

James Degnan

A recent paper by Ebersberger et al. (Molecular Biology and Evolution, 2007) analyzed over 23,000 fragments of DNA for humans, chimpanzees, gorillas, and orangutans. Maximum likelihood can be used to determine which relationships are most compatible with the data (e.g., whether humans are more closely related to chimps or gorillas). Ebersberger et al. (2007) reported the proportions of relationships supported for all trees as well as for highly supported trees.

Trees that are not highly supported often cannot resolve the relationships between humans, chimps, and gorillas. However, ignoring trees that are unresolved leads to different estimates for the proportions of trees supporting different relationships, as well as different estimates for other parameters, such as ancestral population sizes and divergence times. The project I am interested in is simulating genetic DNA on a similar scale with known gene tree distributions to determine the bias introduced by only analyzing well-supported trees. The project would involve a lot of simulation and scripting (use of Perl, Sed and Awk, or similar) in a Linux or Unix environment and working with large data sets. Standard programs can be used for simulation and maximum likelihood estimates.

### **Adaptive sampling designs for environmental science**

Jennifer Brown

Adaptive sampling designs are becoming increasingly popular in environmental science particularly for surveying rare and aggregated populations. An adaptive sample is where the survey design is modified, or adapted, in some way on the basis of information gained during the survey. There are many different adaptive survey designs that can be used to estimate animal and plant abundances. In this project we will review these designs and designs for targeting survey effort to areas of high interest.

## Population genetics

Raazesh Sainudiin

We will study basic Markov processes in coalescent theory from a computationally efficient statistical perspective and characterise the distributions of some classical measures of variation in natural populations. Such classical measures that summarise the data are gaining popularity due to the massive size of current population-genomic data. We will rigorously approach the subject by reading the primary literature, programming, computing, simulating and theorising.

## Probabilities of phylogenetic trees for consensus methods

James Degnan

When evolutionary trees for several species are estimated from different genes (loci), the inferred trees often indicate different relationships. For example, one tree might indicate that humans are more closely related to chimpanzees than gorillas; whereas a second tree might indicate that humans are closer to gorillas than to chimpanzees. When there are multiple trees, a common approach is to use consensus trees, which returns a tree which summarizes the relationships that are most supported by a collection of input trees. Given a model describing the probabilities of different trees, we can compute the probability that a consensus method will return a particular tree. Different consensus methods can result in different trees, and this problem has been worked out for three different consensus methods for trees with four species. Some possible projects are:

- (a) extend the results to trees with five (or more) species
  - (b) consider other consensus methods for trees with four or more species
  - (c) use simulation to examine the effects of mutation models on the probability that a consensus method returns a particular tree.
- (a) and (b) can be done analytically without any simulation, although use of a symbolic

math program such as Maple might be useful.

## Statistics project (extreme values)

Carl Scarrott

Various projects in the area of extreme value statistics are possible. Extreme value theory moves away from the more traditional statistical techniques where the aims are to model the usual (or in some sense average) behaviour, to consider the unusual or rare events. It has received wide application in many fields where the risk associated with rare events is of concern, e.g. finance/economics, hydrological modelling, climate change, engineering (structural design) and material science (material fatigue/failure).

## Projects in Maths or Stats

These projects may be taken either as Mathematics or Statistics projects. It is possible that other projects not listed below may be taken in either area subject to HOD approval.

### Optimization in the presence of noise

Chris Price

This project looks at minimizing a function corrupted by noise. Direct search optimization techniques will be used, which means that gradient information is not available, and might not exist. There is significant flexibility in this project. One can look at a variety of optimization methods, in conjunction with different types of corrupting noise. One could also focus on numerically testing various methods, or on their convergence properties.

Knowledge of MATLAB is highly desirable.

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