Department of Mathematics and Statistics
College of Engineering

UC UNIVERSITY OF CANTERBURY
Te Whare Wānanga o Waitaha
CHRISTCHURCH NEW ZEALAND

Honours 2014

Courses and Projects

14

Department of Mathematics and Statistics

Honours 2014

Welcome to Honours! Honours is an additional year in which you study selected mathematical and/or statistical topics in depth. There is a change of emphasis from the preceding undergraduate years: courses tend to be more focused on a specific problem or class of problems, rather than attempting to give a broad coverage of a branch of Mathematics or Statistics as was done in your earlier undergraduate years. Also in your Honours year, there is the possibility of a project in which you investigate some problem with the assistance of a member of staff. Depending on the nature of the problem, this may involve literature searches in the library, the use of various computing packages (for example MATLAB, Maple or R) on the Departmental computing system, resources on the internet, or even proving new theorems. At the end of the project, you produce a written report and give an oral presentation.

Who should think about Honours? If you view Mathematics and Statistics as more than a means to an end, then the Honours year will be a year well spent. The Honours year provides that additional mental edge, as well as some specific technical and (in the project) presentational skills that potential employers find valuable. If you intend to proceed on to graduate work, Masters or PhD, then Honours is a good stepping-stone.

To enter Honours in Mathematics, you will need at least 60 points from MATH310-399, plus at least 30 points from 300-level MATH, STAT or other approved courses. For Honours in Statistics, you need at least 60 points of 300-level STAT courses and another 30 points from 300-level STAT, MATH or other approved courses. Normally you should have maintained at least a B+ average in these papers. Precise details are given in the University Calendar. There is also a joint Mathematics and Statistics Honours degree (see UC enrolment handbook).

The Department offers both a BA(Hons) and a BSc(Hons). The most appropriate degree is best decided on a case-by-case basis. If you decide that you are interested in Honours, you should see the Head of Department, Prof. Jennifer Brown, or the Honours Coordinator Dr Mark Hickman to discuss your options. You should do this before the enrolment week next year.

Courses are packaged in 20-lecture modules that will run at two lectures a week for a semester. Any proposed programme of study for Honours requires the approval of the Head of Department. It is highly unlikely that any proposed programme that has a high workload in one semester will be approved, so you should try to construct a programme that balances your workload evenly over both semesters.

1. BSc(Hons)

In the Science faculty, Honours may be completed in Mathematics, Statistics, Mathematics and Statistics, Mathematical Physics, Computational and Applied Mathematics (CAMS), Mathematics and Philosophy, Economics and Mathematics, Finance and Mathematics, or, Finance and Statistics.

Students enrolled for a BSc(Hons) in Mathematics and/or Statistics are required to complete eight 400-level courses, as well as a project which is worth the equivalent of two courses.

If you are interested in the:

Mathematical Physics programme, see Prof. David Wiltshire (Physics).

CAMS programme, see Prof. David Wall or Assoc. Prof. Rick Beatson.

Mathematics & Philosophy programme, see Prof. Douglas Bridges or Dr Clemency Montelle.

Economics and Mathematics programme, see Prof. Douglas Bridges or Dr Seamus Hogan (Economics).

Finance and Mathematics programme, see Dr Rua Murray.

Finance and Statistics programme, see Dr Marco Reale.

2. BA(Hons)

In the Arts faculty, Honours may be completed in Mathematics or in Statistics. A BA(Hons) has the same requirements as a BSc(Hons).

3. PG Dip Sci

A Post Graduate Diploma in Science (PGDipSci) can be obtained by completing eight 400-level papers in one year. These courses normally do not include the honours project (MATH/STAT449). Entry requirements are as for a B Sc(Hons), except that you are not required to have a B + average. It is very strongly recommended that your average grade in your majoring subject at stage 3 is at least a C+. A PG Dip Sci may be used as the first part of an MSc.

4. Proposed Courses for 2014

The proposed courses for 2014 are outlined in this booklet. The final decision about which courses are to be offered will depend on the availability of staff and on student interest. The Department reserves the right to cancel any course that does not attract four or more students. It is also possible (and in some cases desirable) to include in your Honours programme appropriate courses from other departments or 300-level courses from Mathematics or Statistics. In the latter case, extra work is usually required to bring the paper to a level consistent with other 400-level courses. Note that, subject to having the required background, any STAT courses may be included in a Mathematics degree and vice versa.

5. Proposed Projects for 2014

A broad range of possible projects is outlined in this booklet. However, this list is not exhaustive and other possibilities for projects are certainly possible. Project supervision is by mutual agreement of the supervisor and student. You should arrange your project by the end of the first week of term in 2014. It is suggested that you seek out possible supervisors during the enrolment week (or before if you have a clear idea of what sort of project interests you). You will hand in a written report on the project in early September; part of your project assessment will be an oral presentation in Term 4.

400-level courses

All 400-level courses are offered subject to sufficient student demand and staff availability, which will be determined at the beginning of each semester. For an up-to-date view of the department's current offerings, please see the department's web page, which is located off the main University web page.

MATHEMATICS

MATH401

Dynamical Systems 1 15 points

MATH401-14S2 (C)

Dynamical systems is a rapidly developing branch of Mathematics with growing applications in diverse fields from traditional areas of applied mathematics to numerical analysis, biological systems, economic models and medicine.

It is often difficult or impossible to write down an exact solution to systems of nonlinear equations. The emphasis in this course will be on qualitative techniques for classifying the behaviour of nonlinear systems, without necessarily solving them exactly. Both main types of dynamical system will be studied: discrete systems, consisting of an iterated map; and continuous systems, consisting of nonlinear differential equations. Topics covered will include: chaotic behaviour of simple 1D maps; period-doubling bifurcations; phase portrait analysis; methods for determining stability of fixed points and limit cycles; centre manifolds; and symbolic dynamics.

Enquiries: Rua Murray

MATH406

Mathematical Models in Biology 15 points

MATH406-14S2 (C)

How did the leopard get its spots? Why should children be vaccinated against measles? This course will try to answer these (and other) questions by using mathematical models to examine biological phenomena.

Some knowledge of dynamical systems and a familiarity with Maple (or MATLAB) are useful pre-requisites to this course.

Enquiries: Alex James

MATH407

Special Topic in Mathematics

(Stochastic Processes) 15 points

MATH407-14S2 (C)

Stochastic processes, in other words process that contain some randomness, are all around us: the price of petrol; a gambler at the casino; a supermarket checkout queue; the weather; world population growth; evolution. This course will look at a variety of stochastic processes and ways in which these can be used to describe the real world. Courses that may be useful include MATH302, MATH363, STAT211. Topics to be chosen from: Poisson processes; birth-death processes; coalescent and branching processes; random walks; gambler's ruin; first hitting times; Markov chains; queuing theory; martingales.

Enquiries: Michael Plank or Mike Steel

MATH410

Approximation Theory 15 points

MATH410-14S1 (C)

Approximation theory lies at the interface of many specialties. As such its study involves an interesting mix of pure and applied mathematics. At the pure end it is the study of the properties certain spaces of functions. Examples being polynomials, splines and Bezier surfaces. At the applied end it is the construction of algorithms to enable efficient use of these spaces of functions in practical problems.

Recent applications of Approximation Theory here at Canterbury include fitting surfaces to noisy point clouds, applied to the custom manufacture of artificial limbs, and fitting geophysical data sets such as gold grade measurement from drill holes in mines.

The first part of this course will concentrate on the fundamentals of approximation of functions of one variable. Central topics will be approximation by algebraic and trigonometric polynomials, and the existence, characterisation and uniqueness of best approximations from finite dimensional normed linear spaces.

In the latter part of the course we will develop some more recent topic. The exact topic will be chosen by the class. Examples of possible topics are the use of Bezier surfaces in modeling, minimum energy interpolation, moving least squares for the approximation of surfaces, penalized least squares and L1 methods for modeling noisy data.

Enquiries: Rick Beatson

MATH412

Unconstrained Optimization 15 points

MATH412-14S1 (C)

This course looks at the minimization of smooth functions of several variables. The first part of the course examines gradient based methods using line searches, including Newton, quasi-Newton, and conjugate gradient methods. A selection of other topics is then introduced, including trust region methods and methods for constrained optimization. Demonstration software is used to illustrate aspects of various algorithms in practice.

Enquiries: Chris Price

MATH416

Exterior Differential Systems 15 points

MATH416-14S1 (C)

Moving frames and exterior differential systems form a natural backdrop for the study of problems in geometry and partial differential equations. In particular, most (if not all) physical systems exhibit symmetry and so embody geometric content in the differential equations that describe the physics. These systems are naturally described in the coordinate free approach of moving frames. Integrability conditions, the "size" of the solution space and the existence of "singular" branches of the solution space for systems of partial differential equations can be readily found in the moving frame approach.

This course will introduce moving frames and exterior differential systems with an emphasis on the conceptual and operational issues. The "standard" vector calculus will be revisited with the aid of differential forms. Their application to simple geometric problems and the reformulation of Maxwell's equations as an exterior differential system will be considered. If time permits (and depending on student interest) either the exterior differential system formulation of Einstein's field equations or application of moving frames to computer recognition of objects will be considered.

Prospective students should have familiarity with partial differential equations and vector calculus.

Enquiries: Mark Hickman

MATH417

Computational Fluid Mechanics **15 points**

MATH417-14S1 (C)

This course is an introduction to the concepts of fluid dynamics and numerical methods for partial differential equations. By combining these concepts the student will be able to carry out numerical experiments involving fluid flows found in aerodynamics and hydrodynamics applications. Topics covered will chosen from: Derivation of basic equations of fluid mechanics. Numerical methods for discretizing partial differential equations; Finite differences, Finite Volumes and Finite Elements. Grid generation. Applications.

Enquiries: Miguel Moyers-Gonzalez or Phil Wilson

MATH420

Hilbert Spaces **15 points**

MATH420-14S2 (C)

The theory of Hilbert spaces is fundamental in many areas of modern mathematical analysis, having a clear and easy-to-grasp geometric structure, just like Euclidean spaces. However, unlike Euclidean spaces, Hilbert spaces may be infinite dimensional. The course will be self-contained, introducing important spaces (especially L^2(m)), operators on them, and basic spectral theory. Prior exposure to MATH343 would be an asset, but is not mandatory.

Enquiries: Douglas Bridges

MATH427

Lie Groups and Lie Algebras **15 points**

MATH427-14S2 (C)

Lie groups are an essential tool in many areas of mathematics and physics. They are often found as groups of symmetries of `nice' mathematical objects like geometries or dynamical systems. The most important Lie groups are finite-dimensional and occur as groups of matrices over real or complex numbers. For example, the group SO(3) of all rotations of Euclidean 3-space or its closely related groups SU(2) and Spin(3) are Lie groups. One is interested in their properties and how these groups can be realised in higher dimensions.

Every Lie group has an associated Lie algebra which is a very good linear approximation of the group. Many properties of the Lie group can be deduced from its Lie algebra.

This course gives an introduction to the basic theory of finite-dimensional Lie groups and their associated Lie algebras and linear representations.

Enquiries: Gunter Steinke

MATH428

Topology 15 points

MATH428-14S1 (C)

Topology, colloquially known as 'rubber-sheet geometry', is the study of continuity in an a bstract setting. Topological notions underpin, or are used in, many areas of mathematics, ranging from analysis to algebraic geometry and even set-theory. Accordingly, the fundamentals of point-set topology are an essential part of the training and armoury of the modern research mathematician. In this course, we introduce topological spaces and study continuity, limits, and may other important notions, in that setting. We then take first steps into uniform spaces, the general setting for notions like uniform continuity and uniform convergence.

The topics will be dr awn from the following: Topological spaces, continuous functions; filters and convergence; compact and c onnected spaces; separation properties; the Stone-Čech compactification; uniform spaces.

Enquiries: Douglas Bridges

MATH429

Combinatorics 15 points

MATH429-14S1 (C)

Matroids (combinatorial geometries) are precisely the structures that underlie the solution of many combinatorial optimization problems. These problems include scheduling and timetabling, and finding the minimum cost of a communications network between cities. Given this, it is perhaps surprising that matroid theory unifies the notions of linear independence in linear algebra and forests in graph theory as well as the notions of duality for graphs and codes. This course is an introduction to matroid theory.

Enquiries: Charles Semple

MATH432

Special Topic in Mathematics (Foundations of Mathematics)

15 points

MATH432-13S2 (C)

An introduction to the philosophy of mathematics, classical and intuitionistic logic, set theory and Gödel's theorems. See MATH 336.

Enquiries: Douglas Bridges

MATH433

Mathematics in Perspective

15 points

MATH433-12S1 (C)

What is Mathematics? What are some of the key moments in the history of Mathematics? What kinds of mathematical result are considered important, and why?

This course is about the history, philosophy, people and major results of Mathematics over the centuries. Since we will m inimise the attention paid to technical details, the course should be accessible not only to those with a 200 level Mathematics background, but also to intellectually mature students in Philosophy and related subjects. In particular, it is strongly recommended for anyone who intends teaching Mathematics at any level from primary school onwards. See MATH380.

Enquiries: John Hannah

MATH439

Topics in Algebra 15 points

MATH439-14S1 (C)

This is a course in modern algebra. See MATH321.

Enquiries: Gunter Steinke

MATH443

Metric, Normed and Hilbert Spaces 15 points

MATH443-14S1 (C)

This course introduces those parts of modern analysis that are essential for many aspects of Pure and Applied Mathematics, Physics, Economics, Finance, and so on. For example, if you want to understand the convergence of numerical algorithms, approximation theory, quantum mechanics, or the economic theory of competitive equilibrium, then you will need to know something about metric, normed and Hilbert spaces. See MATH343.

Enquiries: Qui Bui

MATH449

Project 30 points

MATH449-14W (C)

A whole year research project in mathematics (see Honours projects).

Enquiries: Mark Hickman

MATH491

Summer Research Project 15 points

MATH491-14SU2 (C)

This 150-hour course provides students with an opportunity to develop mathematical research skills and to extend and strengthen their understanding of an area of mathematics.

Enquiries: Gunter Steinke

STATISTICS

STAT445

Multivariate and Financial Time Series 15 points

STAT445-14S2 (C)

Enquiries: Marco Reale

STAT446

Generalised Linear Models 15 points

STAT446-14S1 (C)

How do you analyse data that does not fit the standard methods such as ANOVA and regression? How do you deal with data that are very non-normal, are counts rather than measurements, are correlated and have interdependencies? In this course we introduce you to the very useful toolbox of Generalised Linear Models (GLMs). This is a natural progression from understanding ANOVA, regression and multivariate techniques. We will learn about the general framework for GLMs, and how to use GLMs for analysing data. We will introduce you to the package R, and will use this software throughout the course. Some background in statistical analysis methods is necessary, and useful courses to have completed are STAT220, STAT212 and STAT315. No experience in R is necessary. See STAT319.

Enquiries: Jennifer Brown

STAT449

Project 30 points

STAT449-14W (C)

A whole year research project in Statistics (see Honours projects).

Enquiries: Department of Mathematics and Statistics Reception

STAT450

Advanced Statistical Modelling 15 points

STAT450-14S1 (C)

This course provides an introduction to a range of statistical techniques used in the analysis of spatial data. It will cover the basic concepts and techniques of spatial data analysis (SDA) and pr ovide a wide range of applications examples from various fields such as geology, demographics, epidemiology and environmental sciences. A comprehensive lab programme uses a variety of software packages (including ArcGIS, Geoda, geoR and WinBUGS) to explore and analyse spatial data using the techniques taught in the course.

Enquiries: Elena Moltchanova

STAT455

Sampling Methods 15 points

STAT455-14S1 (C)

This course looks at practical methods for gathering new data, the raw material of Statistics. See STAT312.

Enquiries: Marco Reale

STAT456

Time Series and Stochastic Processes 15 points

STAT456-14S2 (C)

Here we explain some techniques to model observations taken sequentially over time. This kind of data is very common in Biology, Environmental Sciences, Economics and F inance. Time series methods are widely used for forecasting. This course is application oriented, and computers are used to analyse real time series data. See STAT317.

Enquiries: Patrick Saart

STAT460

Extreme Value Statistics 15 points

STAT460-14S1 (C)

This course aims to develop the theory and methods for modelling the extremes of random processes. Extreme value theory moves away from more traditional statistical techniques where the aims are to model the usual (or in some sense average) behaviour, to consider the unusual or rare events. It has received wide application in many fields where the risk associated with rare events are of concern, e.g. finance/economics, hydrological modelling, climate change, engineering (structural design) and material science (material fatigue/failure).

The course will cover the mathematics underlying extreme value models, statistical inference using likelihood and applications to real data, with implementation in the software package R. Recommended preparation includes second year Statistics (preferably STAT214) and at least full first year Mathematics (MATH103 or EMTH119 or equivalent).

Enquiries: Carl Scarrott

STAT 461

Bayesian Inference 15 points

STAT461-14S2 (C)

This course explores the parametric Bayesian approach to Statistics by considering the theory, methods for computing Bayesian solutions, and examples of applications. The key advantage of the Bayesian approach is that it naturally provides probabilistic measures of uncertainty along with the inference. Topics that may be covered include: Theoretical foundations of Bayesian Statistics, choice of prior distributions, Bayesian estimation and credible regions, Bayesian tests and model selection, methods for computing Bayesian solutions, hierarchical models, and applications to linear models. Students should have a sound mathematical background and a good foundation in Statistics and Probability, at least up to the level of STAT213 or STAT214. See STAT314.

Enquiries: Nate Wichitaksom

STAT462

Data Mining 15 points

STAT462-14S2 (C)

Data mining refers to a c ollection of tools to discover patterns and relationships in data, especially for large data bases. It involves several fields including data base management, statistics, artificial intelligence, and machine learning, and it has had a c onsiderable impact in business, industry and science.

This course provides an introduction to the principal methods in data mining: data preparation and warehousing, supervised learning (tree classifiers, neural networks), unsupervised learning (clustering methods), association rules, and the dealing with high-dimensional data (PCA, ICA, multidimensional scaling). Students will see applications from various fields, such as commerce (fraud detection, product placement, targeted marketing, assessing credit risk) and medicine (diagnostics). We will use data mining software to illustrate methods with data sets from these fields.

Students must (i) do problems that are assigned throughout the term and (ii) research an area and write an account of it; the instructor will give suggestions for topics in class. See STAT318.

Enquiries: Nate Wichitaksom

STAT463

Multivariate Statistical Methods 15 points

STAT463-14S1 (C)

Multivariate statistical methods extract information from datasets which consist of variables measured on a number of experimental units. The application of these methods is blooming with the availability of large datasets from a wide range of scientific fields, combined with the advent of computing power to implement them. Examples abound in fields as diverse as bioinformatics, internet traffic analysis, clinical trials, finance and marketing. This course will cover the theory and ap plications of various multivariate statistical methods. See STAT315.

Enquiries: Marco Reale

STAT470

Special Topics in Statistics (Advanced Time Series Methods and Their Applications in Finance, Economics and Science)

15 points

STAT470-14S2 (C)

In many applications, in particular in finance and economics, observed data series often exhibit a behaviour which cannot be modelled with linear time series models (i.e. ARMA processes). Thus alternative models allowing for a nonlinear behaviour are called for and are successfully used. For instance, Robert Engle was awarded the Nobel Prize in 2003 for introducing the so-called (G)ARCH model. In this course we will first review some materials on linear time series methods, then consider and analyse several classes of nonlinear time series models, such as GARCH, Markov-switching as well as threshold autoregressive time series models. We study their common probabilistic and statistical concepts and theory (Markov chains with uncountable state space, stochastic recurrence equations, ergodicity and mixing). Finally, we will derive and apply estimators for the model parameters.

Enquiries: Patrick Saart

STAT472

Special Topic: Advanced Data Analysis and Statistical Consulting

15 points

STAT472-14S1 (C)

In most undergraduate courses, you are taught the theory behind a method and then given neat examples to which it can be applied and software to apply it. In reality, the most common question you will hear from a non-statistician is 'how do I analyse my data? So you are the one who has to come up with the appropriate research question and choose the suitable method (and sometimes learn it quickly too) . It is common in real world applications for the experiments have not been well planned and for data to be missing, which will need to be taken it into account. The assumptions underlying the statistical model (e.g. homoscedasticity and normally distributed) often do not hold and you will have to know what to do. Finally, your fellow scientists, laymen and policymakers are all interested in different aspects of the research question and that is rarely the statistical significance of your ANOVA: you have to know how to communicate your results clearly, correctly and efficiently and how to defend your choices in data analysis and collection.

This course is about the reality of being an applied statistician. Besides covering the above points in class, individual statistical consulting session will provide you with hands-on experience.

Good knowledge of multivariate statistical methods, GLMs, and basic sampling theory expected. Working knowledge of R is recommended.or forecasting methods. I t provides extensive training in forcasting and modelling techniques such as smoothing, dynamic regressions, multivariate autoregressions, state space models, and neural networks with a wide range of applications.

Enquiries: Elena Moltchanova

STAT474

Special topic in Statistics

(Official Statistics) 15 points

STAT474-14S2 (C)

This course provides an overview of the key areas of Official Statistics. Topics covered include data sources (sample surveys and ad ministrative data); the legal and ethical framework of official statistics; an i ntroduction demography; the collection and analysis of health, social and economic data; data visualisation including presentation of spatial data; data matching and i ntegration; the system of National Accounts.

Enquiries: Jennifer Brown

STAT475

Independent course of study

15 points

STAT475-12S1 (C)

STAT475-12S2 (C)

This course allows a s tudent to perform directed reading of a particular topic under a Statistics lecturer. T he topic choice is by mutual arrangement with the lecturer. Students should ensure these arrangements are in place before enrolling in the course.

Enquiries: Any Statistics Lecturer

STAT491

Summer Research Project 15 points

STAT491-13SU2 (C)

This 150-hour course provides students with an opportunity to develop statistical research skills and to extend and strengthen their understanding of an area of statistics.

Enquiries: Gunter Steinke

Honours Projects

A range of possible projects are outlined. However, this list is not exhaustive and other possibilities for projects are certainly possible.

Project supervision is by mutual agreement of the supervisor and student.

You should arrange your project by the end of the first week of term in 2014. It is suggested that you seek out possible supervisors during the enrolment week (or before if you have a clear idea of what sort of project interests you).

You will hand in a written report on the project by Monday September 15, 2014; part of your project assessment will be an oral presentation during Term 4.

Projects in Mathematics

Doing Mathematics Constructively

Douglas Bridges

This project introduces the student to Bishop-style constructive mathematics (BISH), which is, roughly, mathematics with intuitionistic logic and a corresponding set theory. Intuitionistic logic, as distinct from the ‘classical’ logic that we normally use in mathematics courses and research, captures the idea that in order to prove that ‘there exists x with the property P’, we should provide algorithms for constructing x and then showing that P(x) holds. Exclusive use of that logic enables us to develop algebra, analysis, topology, ... in a pur ely constructive/algorithmic fashion, and reveals distinctions that are hidden by classical logic.

Once we have covered the fundamentals of constructive thinking, the student will examine, within BISH, some aspects of algebra, analysis, or topology (depending on the student's preference). By the end of the project, the student should have a good grasp of the differences, in theory and in practice, between constructive and non-constructive methods, and will have experienced a view of algebra, analysis, or topology that is quite different to that gained in standard lecture courses.

Algorithms for tangent plane continuous surface fitting

Rick Beatson

This project considers the fitting of tangent plane continuous surfaces to points and normals data sets. The end goal is to efficiently model underground rock formations, and other blobby shapes, occurring in applications. The project will build on an existing algorithm which uses quintic Bezier elements.

There are diverse aspects that could be considered. An algorithmic aspect is to consider and experiment with some local optimisation steps, which would be used to improve a given fit when the normals are noisy.

Another aspect is to recast the current theory in terms of a different parametrisation of the Bezier elements. This would quite probably give rise to a cleaner theory, and probably a more efficient implementation.

Finally, on the implemtation end, a core task in the current code is to solve a sparse semi-definite system. The current solver is crude. The theory and implementation of a more efficient solver would be an interesting task for someone who enjoys numerical analysis, and wants to experiment with parallel algorithms.

Geodesics in semi-Riemannian Manifolds

Gunter Steinke

What are the “straight lines” on a sphere? To answer this question one has to make precise what ”straight lines” in a curved space mean. First of all, spheres are manifolds. Manifolds are generalisations of

Euclidean spaces in that they only look locally like Euclidean spaces. They emerge in a variety of mathematical or physical contexts and comprise many well-known examples like spheres, tori, curved surfaces in 3-space or space-time in physics. Often manifolds carry an additional structure that allows one to measure the speed of particles moving in it. This leads to the general concept of semi-Riemannian manifolds.

Straight line segments in Euclidean space are shortest curves between two points. The role of straight lines in semi-Riemannian manifolds is played by geodesics; they are locally shortest curves but can exhibit interesting global behaviour, like self-intersection in some Riemannian manifolds. Geodesics are of significance among others in physics, since they describe, for example, the path light travels in the universe or a free-falling particle is moving along. Geodesics can described in local coordinates by certain second order partial differential equations.

The project investigates manifolds and in particular semi-Riemannian manifolds, and explores the notion of geodesics therein. It is aimed to gain an understanding and appreciation of (semi-Riemannian) manifolds and their geometry, and further to prove the so-called Hopf-Rinow Theorem, which deals with geodesical completeness of Riemannian manifolds and relates existence of geodesics between any two points to other topological properties of the manifold. The project also looks at geodesics in some low-dimensional semi-Riemannian manifolds and attempts to obtain complete descriptions of their geodesics.

History of Mathematics

John Hannah

The second half of the 19th century and the first half of the 20th century saw the introduction and development of several quite abstract branches of mathematics. Examples include abstract algebra (groups, fields, rings, ideals, modules), functional analysis, topology, and category theory. In this project you will follow this development in a topic of your own choosing, and in the process you will both learn some new mathematics, and try to discern the influences (mathematical, philosophical or cultural in origin) which led to these developments.

Reflection Groups

Gunter Steinke

Groups naturally occur as collections of symmetries of geometries or algebraic structures. Of all symmetries involutions, that is, symmetries of order 2, often play a special role, and many important groups, like symmetric groups, are known to be generated by involutions.

The project investigates groups that are generated by finitely many involutions. It should lead to a complete classification of some of the so-called Coxeter groups. Much of the information about such groups can be graphically represented in Coxeter-Dynkin diagrams. Similar diagrams also appear in seemingly unrelated areas like Lie algebras, algebraic groups and buildings. Furthermore, realisations of the finite Coxeter groups as reflection groups, that is, groups of isometries in some Euclidean space generated by reflections, should be found.

Negative Correlation in Graphs

Charles Semple

It follows from the work of Kirchhoff (1847) that the spanning trees of a connected graph satisfy the negative correlation inequality. In particular, if T is a spanning tree of a connected graph G chosen uniformly at random, then the probability that T contains an edge e given that T contains an edge f is less than or equal to the probability that T contains e for all distinct edges e and f of G. In other words, the events `T contains e' and the event `T contains f' are negatively correlated. This project investigates the question of whether or not the analogous result holds for the forests of a connected graph G.

Lax Pairs

Mark Hickman

Given a non-linear differential equation, a *Lax pair* is a pair of *linear* differential operators $\mathcal{L}$, $\mathcal{M}$ whose commutator vanishes only on solutions of the differential equation. A Lax pair allows one to potentially solve the differential equation by reducing the problem to an eigenvalue problem (if the operator $\mathcal{L}$ is second order, this is a Sturm-Liouville problem) and a time evolution of the eigenfunction; the so-called inverse scattering method. If $\mathcal{L}$ is first order then the Lax pair gives a conservation law of the differential equation. In this project, we will be looking at a method to compute the Lax pair of prescribed order for a differential equation (if it exists). This will involve Maple and would suit a student who has completed MATH302.

Modelling Collective Cell Movement

Mike Plank

This project will build mathematical models of cell invasion based on random walks. A key advantage of this type of model is that it can be simulated using simple computer programs, allowing a wide range of different scenarios to be efficiently investigated. Each individual cell in the model moves stochastically, for example by selecting a movement direction at random. Despite the unpredictability of an individual cell's movement, the model can be used to predict the emergent behavior of the population as a whole, for example the number of cells at a particular location, with a surprising degree of accuracy. One goal of this project is to develop and solve a PDE to describe how the average cell density changes over space and time.

Calcium Waves

Mike Plank

Smooth muscle cells use the concentration of calcium inside the cell to regular a variety of functions. The calcium dynamics within an individual cell can be modelled by a nonlinear dynamical system. But cells do not exist in isolation, they communicate with neighbouring cells. This can lead to the propagation of waves of calcium along a series of interconnected cells. This project will investigate the behaviour of these waves using a mathematical model. The model will use ideas from dynamical systems, PDEs and numerical methods. One goal of

this project is to understand how certain features of the calcium waves, such as wave speed, direction and frequency, depend on the parameters that govern the individual cell behaviour.

Ecological Networks

Alex James

From the Amazon rainforest to the Okeover stream species interact with each other in complex systems. These interactions come in many forms: competitive - they both compete for the same resource, predator-prey - eat or be eaten, mutualistic - both benefit from the relationship. How does the architecture of these interactions affect the behaviour of the system? Should a species interact with as many others as possible?

Should they form groups or cliques? Is there a conflict between good for an individual and good for the system?

Useful pre-requisites for this project could include dynamical systems, discrete maths and some Matlab or other computer programming. An interest in ecology is also helpful.

I also have other projects available that combine maths and ecology. Please contact me to find out more.

Projects in Statistics

Extreme Value Statistical Modelling

Carl Scarrott

The statistics you meet on undergraduate course typically focus on the capturing the "usual" characteristics of a process (e.g. properties of mean, median or variance). Extreme value statistics focus on understanding the unusual or rare events of a process. Extremes are of interests in all sort of different fields, for example:

1) financial risk (e.g. estimating Value at Risk for risk and portfolio management)

2) engineering (e.g. designing structures to withstand the stongest forces they could be exposed to, e.g. wind exposure on bridges or forces on buildings due to earthquakes!)

3) environment (e.g. are Christchurch's winter air pollution extremes reducing due to government interventions?)

4) IT (e.g. can our servers and network cope with peak demand when your assignments and projects are due in?)

I have various honours projects in extreme value modelling for financial, environmental, medical and industrial applications. So if you are thinking of becoming a statistical "extremist" then get in touch

Environmental Statistics

Jennifer Brown

Environmental monitoring is a fast moving, and important field of research. Data on environmental processes such as changes in water quality, endangered species distribution, weed invasion, and biodiversity are used to inform and guide how we manage our environment.

One use of environmental data is to build models to predict species distribution, and to predict the effect of environmental changes. In this project we will look at different methods used to collect field data and the effect of these differences on prediction models. We will use computer simulations to model data collection and analysis.

Bayesian Econometrics of Financial Time Series Models

Nate Wichitaksom

This project aims to equip you with a Bayesian method to analyze some financial time series models of your choice. Through the project, you will learn how Bayesian algorithms can effectively and efficiently estimate model parameters. In addition, fresh experience from real data examples on stock returns will further your understanding on the applicability of your work.

Geospatial Analysis

Elena Moltchanova

The aim of the project will be to demonstrate application of spatial statistical techniques in practice. A suitable dataset may either be selected from the field of interest by the student or will be supplied by the lecturer. A short report should then be produced with the short description of the data, some results of the exploratory data analysis and, finally, the description of the method of choice and the results of its application.

University of Canterbury
Te Whāre Wananga o Waitaha
Private Bag 4800
Christchurch 8140
New Zealand

Telephone: +64 366 7001
Freephone in NZ: 0800 VARSITY (0800 827 748)
Facsimile: +64 3 364 2999
Email: info@canterbury.ac.nz
www.canterbury.ac.nz