School of Mathematics and Statistics
College of Engineering

UNIVERSITY OF CANTERBURY
Te Whare Wānanga o Waitaha
CHRISTCHURCH NEW ZEALAND

Honours 2016

Courses and Projects

16

School of Mathematics and Statistics

Honours 2016

Welcome to Honours! Honours is an additional year in which you study selected mathematical and/or statistical topics in depth. There is a change of emphasis from the preceding undergraduate years: courses tend to be more focused on a specific problem or class of problems, rather than attempting to give a broad coverage of a branch of Mathematics or Statistics as was done in your earlier undergraduate years.

Also in your Honours year, there is the possibility of a project in which you investigate some problem with the assistance of a member of staff. Depending on the nature of the problem, this may involve literature searches in the library, the use of various computing packages (for example Matlab, Maple or R) on the school's computing system, resources on the internet, or even proving new theorems. At the end of the project, you produce a written report and give an oral presentation.

Who should think about Honours? If you view Mathematics and Statistics as more than a means to an end, then the Honours year will be a year well spent. The Honours year provides that additional mental edge, as well as some specific technical and (in the project) presentational skills that potential employers find valuable. If you intend to proceed on to graduate work, Masters or PhD, then Honours is a good stepping-stone.

To enter Honours in Mathematics, you will need at least 60 points from MATH310-399, plus at least 30 points from 300-level MATH, STAT or other approved courses. For Honours in Statistics, you need at least 60 points of 300-level STAT courses and another 30 points from 300-level STAT, MATH or other approved courses. Normally you should have maintained at least a B+ average in these papers. Precise details are given in the University Calendar. There is also a joint Mathematics and Statistics Honours degree (see UC enrolment handbook).

The School offers both B.A.(Hons), B.Sc.(Hons) and a P.G.Dip.Sc. programs. The most appropriate program is best decided on a case-by-case basis. If you decide that you are interested in Honours, you should see the Head of School, Prof. Jennifer Brown, or the Honours Coordinator Dr Mark Hickman to discuss your options. You should do this before the enrolment week next year.

Courses are packaged in modules that will run at two lectures a week for a semester. Any proposed programme of study for Honours requires the approval of the Head of School. It is highly unlikely that any proposed programme that has a high workload in one semester will be approved, so you should try to construct a programme that balances your workload evenly over both semesters. Courses at the appropriate level from other departments may be included in an approved program. In addition there are a number of joint programs between the School of Mathematics and Statistics and other departments at the university.

B.Sc.(Hons)

In the Science faculty, Honours may be completed in Mathematics, Statistics, Mathematics and Statistics, Mathematical Physics, Computational and Applied Mathematics (CAMS), Mathematics and Philosophy or Economics and Mathematics. Students enrolled for a B.Sc.(Hons) in Mathematics and/or Statistics are required to complete eight 400-level courses, as well as a project which is worth the equivalent of two courses. If you are interested in the:

- Mathematical Physics programme, see Prof. David Wiltshire (Physics).
- CAMS programme, see Assoc. Prof. Rick Beatson.
- Mathematics and Philosophy programme, see Dr. Maarten McKubre-Jordens or Dr Clemency Montelle.
- Economics and Mathematics programme, see Dr. Rua Murray.

B.A.(Hons)

In the Arts faculty, Honours may be completed in Mathematics or in Statistics. A B..A. (Hons) has the same requirements as a B.Sc.(Hons).

P.G.Dip.Sci.

A Post Graduate Diploma in Science (P.G.Dip.Sci.) can be obtained by completing eight 400-level papers in one year. These courses normally do not include the honours project (MATH/STAT449). Entry requirements are as for a B.Sc.(Hons) except that you are not required to have a B+ average. It is very strongly recommended that your average grade in your majoring subject at stage 3 is at least a C+. A P.G.Dip.Sci may be used as the first part of an M.Sc.

Proposed Courses for 2016

The proposed courses for 2016 are outlined in this booklet. The final decision about which courses are to be offered will depend on the availability of staff and on student interest. The School reserves the right to cancel any course that does not attract four or more students. It is also possible (and in some cases desirable) to include in your Honours programme appropriate courses from other departments or 300-level courses from Mathematics or Statistics. In the latter case, extra work is usually required to bring the paper to a level consistent with other 400-level courses. Note that, subject to having the required background, any STAT courses may be included in a Mathematics degree and vice versa.

Proposed Projects for 2016

A broad range of possible projects is outlined in this booklet. However, this list is not exhaustive and other possibilities for projects are certainly possible. Project supervision is by mutual agreement of the supervisor and student. You should arrange your project by the end of the first week of term in 2016. It is suggested that you seek out possible supervisors during the enrolment week (or before if you have a clear idea of what sort of project interests you). You will hand in a written report on September 12; part of your project assessment will be an oral presentation in Term 4.

400-LEVEL COURSES

All 400-level courses are offered subject to sufficient student demand and staff availability, which will be determined at the beginning of each semester. For an up-to-date view of the School's current offerings, please see the School's web page, which is located off the main University web page.

Mathematics

MATH401 **15 points**
Dynamical Systems 1
MATH401-16S1 (C)

Dynamical systems is a rapidly developing branch of Mathematics with growing applications in diverse fields from traditional areas of applied mathematics to numerical analysis, biological systems, economic models and medicine.

It is often difficult or impossible to write down an exact solution to systems of nonlinear equations. The emphasis in this course will be on qualitative techniques for classifying the behaviour of nonlinear systems, without necessarily solving them exactly. Both main types of dynamical system will be studied: discrete systems, consisting of an iterated map; and continuous systems, consisting of nonlinear differential equations. Topics covered will include: chaotic behaviour of simple 1D maps; period-doubling bifurcations; phase portrait analysis; methods for determining stability of fixed points and limit cycles; centre manifolds; and symbolic dynamics.

Enquires: Rua Murray

MATH406 **15 points**
Mathematical Models in Biology
MATH406-16S1 (C)

How did the leopard get its spots? Why should children be vaccinated against measles? This course will try to answer these (and other) questions by using mathematical models to examine biological phenomena. In the achieving this we will study: Biochemical reactions, reaction diffusion, cellular homeostasis, membrane ion channels, excitability and nonlinear wave propagation.

Some knowledge of dynamical systems and a familiarity with Maple (or Matlab) are useful pre-requisites to this course.

Enquires: Alex James or Phil Wilson

MATH407 **15 points**
Special Topic in Mathematics (Stochastic Processes)
MATH407-16S2 (C)

Term 3: (Mike Steel) Probability theory Random graphs; probabilistic method Branching processes, Polya-urn models, martingales

Term 4: (Michael Plank) Discrete random

walks Continuous random walks Poisson and pure-birth processes Markov processes

Enquires: Michael Plank or Mike Steel

MATH409 15 points
Cryptography and Coding Theory II
MATH409-16S1 (C)

The purpose of this course is to study applications of Number Theory to Cryptography and Coding Theory. These topics address the problem of preserving data integrity during transmission or storage against malicious attacks, in the first case and against corruption due to random noise in the second. The main ideas will be introduced and then methods to perform efficiently the number theoretic calculations that come up in cryptography and coding theory will be studied. In particular, we will cover the use of elliptic curves in cryptography.

Enquires: Felipe Voloch

MATH410 15 points
Approximation Theory
MATH410-16S1 (C)

Approximation theory lies at the interface of many specialties. As such its study involves an interesting mix of pure and applied mathematics. At the pure end it is the study of the properties certain spaces of functions. Examples being polynomials, splines radial basis functions and Bezier surfaces. At the applied end it is the construction of algorithms to enable efficient use of these spaces of functions in practical problems.

Recent applications of Approximation Theory here at Canterbury include fitting surfaces to noisy point clouds, applied to the custom manufacture of artificial limbs, and fitting geophysical data sets such as gold grade measurements from drill holes in mines.

The first part of this course will concentrate on the fundamentals of approximation of functions of one variable. Central topics will be approximation by algebraic and trigonometric polynomials, and the existence, characterisation and uniqueness of best approximations from finite dimensional normed linear spaces.

In the latter part of the course we will develop some more recent topic. The exact topic will be chosen by the class. Examples of possible topics are radial basis functions, the use of Bezier surfaces in modeling, and penalized least squares and L1 methods for modeling noisy data.

Enquires: Rick Beatson

MATH411 15 points
Topics in Algebra
MATH411-16S1 (C)

This course will provide an introduction to Algebraic Geometry. This is the study of solutions to systems of polynomial equations, for example a curve in the plane defined by an equation $x^3+y^3 = 1$. The methods are mostly algebraic, but the intuition comes from geometry. You will develop your ability to translate your geometric intuition (which tells you what you should be expecting) into precise algebraic expressions (which allow you to cash in on that intuition). Topics covered will include algebraic sets in affine and projective space, algebraic curves, elliptic curves, curve singularities, and others as time permits.

Familiarity with linear and abstract algebra (MATH203 and MATH321) is encouraged.

Enquires: Brendan Creutz

MATH412 **15 points**
Unconstrained Optimization
MATH412-16S2 (C)

This course looks at the minimization of smooth functions of several variables. The first part of the course examines gradient based methods using line searches, including Newton, quasi-Newton, and conjugate gradient methods. A selection of other topics is then introduced, including trust region methods and methods for constrained optimization. Demonstration software is used to illustrate aspects of various algorithms in practice.

Enquires: Rachael Tappenden

MATH420 **15 points**
Hilbert Spaces
MATH420-16S2 (C)

The theory of Hilbert spaces is fundamental in many areas of modern mathematical analysis, having a clear and easy-to-grasp geometric structure, just like Euclidean spaces. However, unlike Euclidean spaces, Hilbert spaces may be infinite dimensional. The course will be self-contained, introducing important spaces (especially L2(m)), operators on them, and basic spectral theory. Prior exposure to MATH343 would be an asset, but is not mandatory.

Enquires: Maarten McKubre-Jordens or Hannes Diener

MATH427 **15 points**
Lie Groups and Lie Algebras
MATH427-16S2 (C)

Lie groups are an essential tool in many areas of mathematics and physics. They are often found as groups of symmetries of 'nice' mathematical objects like geometries or dynamical systems. The most important Lie groups are finite-dimensional and occur as groups of matrices over real or complex numbers. For example, the group SO(3) of all rotations of Euclidean 3-space or its closely related groups SU(2) and Spin(3) are Lie groups. One is interested in their properties and how these groups can be realised in higher dimensions.

Every Lie group has an associated Lie algebra which is a very good linear approximation of the group. Many properties of the Lie group can be deduced from its Lie algebra.

This course gives an introduction to the basic theory of finite-dimensional Lie groups and their associated Lie algebras and linear representations.

Enquires: Gunter Steinke

MATH429 **15 points**
Combinatorics
MATH429-16S2 (C)

Matroids (combinatorial geometries) are precisely the structures that underlie the solution of many combinatorial optimisation problems. These problems include scheduling and timetabling, and finding the minimum cost of a communications network between cities. Matroid theory also unifies the notions of linear independence in linear algebra and forests in graph theory as well as the notions of duality for graphs and codes. This course is an introduction to matroid theory and is designed for mathematics and computer science students.

Enquires: Charles Semple

MATH431 **15 points**
Graph Theory
MATH431-16S2 (C)

In a nutshell, graphs are mathematical structures which model relationships between objects. Graph theory is the branch of combinatorics concerned with their study, and has grown to become a very rich and diverse discipline in its own right. It has applications in almost every scientific field, from analysing the spread of epidemics to modelling social networks.

In this self-contained course we will explore a range of topics from graph theory, considering both theory and applications. The course is intended for students majoring in Mathematics or Computer Science. Does not require MATH120 or MATH220.

Enquires: Jeanette McLeod

MATH432 **15 points**
Special Topic in Mathematics (Foundations of Mathematics)
MATH432-16S2 (C)

An introduction to the philosophy of mathematics, classical and intuitionistic logic, set theory and Gödel's theorems. See MATH 336.

Enquires: Maarten McKubre-Jordens

MATH433 **15 points**
Mathematics in Perspective
MATH433-16S1 (C)

What is Mathematics? What are some of the key moments in the history of Mathematics? What kinds of mathematical result are considered important, and why?

This course is about the history, philosophy, people and major results of Mathematics over the centuries. Since we will minimise the attention paid to technical details, the course should be accessible not only to those with a 200 level Mathematics background, but also to intellectually mature students in Philosophy and related subjects. In particular, it is strongly recommended for anyone who intends teaching Mathematics at any level from primary school onwards. See MATH380.

Enquires: John Hannah

MATH438 **15 points**
Special Topic in Mathematics (Advanced Complex Variables
MATH438-16S2 (C)

This is a second course in complex variables that introduces the student to some beautiful results and important applications of complex analysis. Topics covered include: Liouville's theorem, open mapping theorem, Cassorati-Weierstrass theorem, argument principle, Rouche's theorem, maximum modulus principle, Schwarz's lemma, normal families, Riemann mapping theorem. Additional material (time permitting) may be chosen from the following: Infinite products, Runge's theorem, univalent functions, subharmonic functions, value distribution theory, Hardy spaces.

Enquires: Ngin-Tee Koh

MATH439 **15 points**
Topics in Algebra
MATH439-16S1 (C)

This course in modern algebra. See MATH321.

Enquires: Gunter Steinke

MATH443 **15 points**
Metric, Normed and Hilbert Spaces

MATH443-16S1 (C)

This course introduces those parts of modern analysis that are essential for many aspects of Pure and Applied Mathematics, Physics, Economics, Finance, and so on. For example, if you want to understand the convergence of numerical algorithms, approximation theory, quantum mechanics, or the economic theory of competitive equilibrium, then you will need to know something about metric, normed and Hilbert spaces. See MATH343.

Enquires: Hannes Diener

MATH449 **30 points**
Project
MATH449-16W (C)

A whole year research project in mathematics (see Honours projects).

Enquires: Mark Hickman

MATH475 **15 points**
Independent course of study
MATH475-16S1 or MATH475-16S2 (C)

This course allows a student to perform directed reading of a particular topic under a Mathematics lecturer. The topic choice is by mutual arrangement with the lecturer. Students should ensure these arrangements are in place before enrolling in the course.

Enquires: Any Mathematics Lecturer

MATH491 **15 points**
Summer Research Project
MATH491-16SU2 (C)

This 150-hour course provides students with an opportunity to develop mathematical research skills and to extend and strengthen their understanding of an area of mathematics.

Enquires: Jeanette McLeod

Statistics

STAT446 — 15 points
Generalised Linear Models
STAT446-16S1 (C)

How do you analyse data that does not fit the standard methods such as ANOVA and regression? How do you deal with data that are very non-normal, are counts rather than measurements, are correlated and have interdependencies? In this course we introduce you to the very useful toolbox of Generalised Linear Models (GLMs). This is a natural progression from understanding ANOVA, regression and multivariate techniques. We will learn about the general framework for GLMs, and how to use GLMs for analysing data. We will introduce you to the package R, and will use this software throughout the course. Some background in statistical analysis methods is necessary, and useful courses to have completed are STAT220, STAT212 and STAT315. No experience in R is necessary. See STAT319.

Enquires: Jennifer Brown

STAT447 — 15 points
(Official Statistics)
STAT447-16S2 (C)

This course provides an overview of the key areas of Official Statistics. Topics covered include data sources (sample surveys and administrative data); the legal and ethical framework of official statistics; an introduction demography; the collection and analysis of health, social and economic data; data visualisation including presentation of spatial data; data matching and integration; the system of National Accounts.

Enquires: Jennifer Brown

STAT449 — 30 points
Project
STAT449-16W (C)

A whole year research project in Statistics (see Honours projects).

Enquires: School of Mathematics and Statistics Reception

STAT450 — 15 points
Advanced Statistical Modelling
STAT450-16S1 (C)

This course provides an introduction to a range of statistical techniques used in the analysis of spatial data. It will cover the basic concepts and techniques of spatial data analysis (SDA) and provide a wide range of applications examples from various fields such as geology, demographics, epidemiology and environmental sciences. A comprehensive lab programme uses a variety of software packages (including ArcGIS, Geoda, geoR and WinBUGS) to explore and analyse spatial data using the techniques taught in the course.

Enquires: Elena Moltchanova

STAT455 — 15 points
Sampling Methods
STAT455-16S1 (C)

This course looks at practical methods for gathering new data, the raw material of Statistics. See STAT312.

Enquires: Blair Robertson

STAT456 **15 points**
Time Series and Stochastic Processes
STAT456-16S2 (C)

Here we explain some techniques to model observations taken sequentially over time. This kind of data is very common in Biology, Environmental Sciences, Economics and Finance. Time series methods are widely used for forecasting. This course is application oriented, and computers are used to analyse real time series data. See STAT317.

Enquires: Marco Reale

STAT460 **15 points**
Extreme Value Statistics
STAT460-16S1 (C)

This course aims to develop the theory and methods for modelling the extremes of random processes. Extreme value theory moves away from more traditional statistical techniques where the aims are to model the usual (or in some sense average) behaviour, to consider the unusual or rare events. It has received wide application in many fields where the risk associated with rare events are of concern, e.g. finance/economics, hydrological modelling, climate change, engineering (structural design) and material science (material fatigue/failure).

The course will cover the mathematics underlying extreme value models, statistical inference using likelihood and applications to real data, with implementation in the software package R. Recommended preparation includes second year Statistics (preferably STAT214) and at least full first year Mathematics (MATH103 or EMTH119 or equivalent).

Enquires: Carl Scarrott

STAT461 **15 points**
Bayesian Inference
STAT461-16S2 (C)

This course explores the parametric Bayesian approach to Statistics by considering the theory, methods for computing Bayesian solutions, and examples of applications. The key advantage of the Bayesian approach is that it naturally provides probabilistic measures of uncertainty along with the inference. Topics that may be covered include: Theoretical foundations of Bayesian Statistics, choice of prior distributions, Bayesian estimation and credible regions, Bayesian tests and model selection, methods for computing Bayesian solutions, hierarchical models, and applications to linear models. Students should have a sound mathematical background and a good foundation in Statistics and Probability, at least up to the level of STAT213 or STAT214. See STAT314.

Enquires: Elena Moltchanova

STAT462 **15 points**
Data Mining
STAT462-16S2 (C)

Data mining refers to a collection of tools to discover patterns and relationships in data, especially for large data bases. It involves several fields including data base management, statistics, artificial intelligence, and machine learning, and it has had a considerable impact in business, industry and science.

This course provides an introduction to the principal methods in data mining: data preparation and warehousing, supervised learning (tree classifiers, neural networks), unsupervised learning (clustering methods), association rules, and the dealing with high-dimensional data (PCA, ICA, multidimensional scaling). Students will see applications from various fields, such as commerce (fraud detection, product placement, targeted marketing, assessing credit risk) and medicine (diagnostics). We will use data mining software to illustrate methods with data sets from these fields.

Students must (i) do problems that are assigned throughout the term and (ii) research an area and write an account of it; the instructor will give suggestions for topics in class. See STAT318.

Enquires: Blair Robertson

STAT463 15 points
Multivariate Statistical Methods
STAT463-16S2 (C)

Multivariate statistical methods extract information from datasets which consist of variables measured on a number of experimental units. The application of these methods is blooming with the availability of large datasets from a wide range of scientific fields, combined with the advent of computing power to implement them. Examples abound in fields as diverse as bioinformatics, internet traffic analysis, clinical trials, finance and marketing. This course will cover the theory and applications of various multivariate statistical methods. See STAT315.

Enquires: Daniel Gerhard

STAT470 15 points
Special Topics in Statistics
(Advanced Time Series Methods)
STAT470-16S2 (C)

In many applications, in particular in finance and economics, observed data series often exhibit a behaviour which cannot be modelled with linear time series models (i.e. ARMA processes). Thus alternative models allowing for a nonlinear behaviour are called for and are successfully used. For instance, Robert Engle was awarded the Nobel Prize in 2003 for introducing the so-called (G)ARCH model. In this course we will first review some materials on linear time series methods, then consider and analyse several classes of nonlinear time series models, such as GARCH, Markov-switching as well as threshold autoregressive time series models. We study their common probabilistic and statistical concepts and theory (Markov chains with uncountable state space, stochastic recurrence equations, ergodicity and mixing). Finally, we will derive and apply estimators for the model parameters.

Enquires: Marco Reale

STAT471 15 points
Special Topic in Statistics
(Big Data)
STAT471-16S1 (C)

Big Data refers to the large and often complex datasets generated in the modern world: data sources such as commercial customer records, internet transactions, environmental monitoring. This course provides an introduction to the theory and practice of working with Big Data and will be of interest to any student wanting to prepare themselves for work industry, or further research.

The course will give you an understanding of the basic concepts of how and where Big Data arises. By taking the course you will be able to describe data structures and mechanisms for the capture, storage, processing, summary and visualisation of Big Data; implement practical methods for data acquisition and management using appropriate software (SQL, R, Python, Perl; Hadoop, Mapreduce); and, understand basic methods of analysis of Big Data, including methods from machine learning for high dimensional data.

Enquires: Jennifer Brown

STAT472 15 points
Special Topic in Statistics (Advanced Data Analysis and Statistical Consulting)
STAT472-16S2 (C)

In most undergraduate courses, you are taught the theory behind a method and then given neat examples to which it can be applied and software to apply it. In reality, the most common question you will hear from a non-statistician is 'how do I analyse my data? So you are the one who has to come up with the appropriate research question and choose the suitable method (and sometimes learn it quickly too) . It is common in real world applications for the experiments have not been well planned and for data to be missing, which will need to be taken it into account. The assumptions underlying the statistical model (e.g. homoscedasticity and normally distributed) often do not hold and you will have to know what to do. Finally, your fellow scientists, laymen and policy-makers are all interested in different aspects of the research question and that is rarely the statistical significance of your ANOVA: you have to know how to communicate your results clearly, correctly and efficiently and how to defend your choices in data analysis and collection.

This course is about the reality of being an applied statistician. Besides covering the above points in class, individual statistical consulting session will provide you with hands-on experience.

Good knowledge of multivariate statistical methods, GLMs, and basic sampling theory expected. Working knowledge of R is recommended.or forecasting methods. It provides extensive training in forcasting and modelling techniques such as smoothing, dynamic regressions, multivariate autoregressions, state space models, and neural networks with a wide range of applications.

Enquires: Elena Moltchanova

STAT474 15 points
Special Topic in Statistics (Scalable Data Science)
STAT474-16S1 (C)

Scalable data science is all about analysing 'big data', real-world datasets that do not fit into any single computer. So one needs a cloud-computing platform, a managed distributed computing environment made up of a network of several tens or hundreds of computers, to analyse such big data. This advanced undergraduate course involves a flipped class-room of the archived edX course on scalable machine learning. These edX lectures and the accompanying labs using Apache Spark will be supplemented by weekly tutorials and discussions of research papers underpinning the core ideas of Spark. This independent course of study can form the groundwork for an honours project in scalable data science that is customized to the interests and skills of the student.

More details can be found here: `http://www.math.canterbury.ac.nz/~r.sainudiin/courses/ScalableDataScience/`

Enquires: Raazesh Sainudiin

STAT475 **15 points**
Independent course of study
STAT475-16S1 or STAT475-16S2 (C)

This course allows a student to perform directed reading of a particular topic under a Statistics lecturer. The topic choice is by mutual arrangement with the lecturer. Students should ensure these arrangements are in place before enrolling in the course.

Enquires: Any Statistics Lecturer

STAT491 **15 points**
Summer Research Project
STAT491-16SU2 (C)

This 150-hour course provides students with an opportunity to develop statistical research skills and to extend and strengthen their understanding of an area of statistics.

Enquires: Jeanette McLeod

Honours Projects

A range of possible projects are outlined. However, this list is not exhaustive and other possibilities for projects are certainly possible.

Project supervision is by mutual agreement of the supervisor and student. You should arrange your project by the end of the first week of term in 2016. It is suggested that you seek out possible supervisors during the enrolment week (or before if you have a clear idea of what sort of project interests you).

You will hand in a written report on the project by Monday September 15, 2016; part of your project assessment will be an oral presentation during Term 4.

Projects in Mathematics

Algorithms for tangent plane continuous surface fitting
Rick Beatson

This project considers the fitting of tangent plane continuous surfaces to points and normals data sets. The end goal is to efficiently model underground rock formations, and other blobby shapes, occuring in applications. The project will build on an existing algorithm which uses quintic Bezier elements.

There are diverse aspects that could be considered. An algorithmic aspect is to consider and experiment with some local optimisation steps, which would be used to improve a given fit when the normals are noisy.

Another aspect is to recast the current theory in terms of a different parametrisation of the Bezier elements. This would quite probably give rise to a cleaner theory, and probably a more efficient implementation.

Finally, on the implementation end, a core task in the current code is to solve a sparse semi-definite system. The current solver is crude. The theory and implementation of a more efficient solver would be an interesting task for someone who enjoys numerical analysis, and wants to experiment with parallel algorithms.

Arithmetic of Elliptic Curves
Brendan Creutz

An elliptic curve is a geometric object with a rich algebraic structure. There is a natural way to 'add' two points on the curve to get a third, resulting in an algebraic group. One of the most prominent open problems in mathematics seeks to obtain a better understanding of the structure of such groups (cf. The Birch and Swinnerton-Dyer Conjecture at `http://www.claymath.org/millennium-problems`).

These groups have also begun to play an increasing prevalent role in cryptography, as they provide cryptosystems that are more secure and efficient than traditional systems based on integer factorization or discrete logarithms in finite fields.

This project will introduce the student to the theory of elliptic curves by first investigating the classical geometry of elliptic curves over the real or complex numbers. Depending on student interest, the project will then explore the situation over the rational numbers, function fields, or finite fields, as well as investigating potential applications.

A Mathematical Approach to Optimal Filtering
Miguel Moyers Gonzalez, Rua Murray and Phil Wilson

Porous media are materials composed of a network of pores in a solid matrix. They are important for a large range of real life applications in the oil, gas and process industries, structured lightweight materials, and studies of flow properties. 3D printers offer the opportunity to control the size, shape and location of the voids in the porous morphology. The goal of the project is to develop and study a mathematical approach to the different geometrical morphologies that can be theoretically designed in porous media. The problem can be formulated as an optimal control problem and will be studied using MATLAB or an equivalent.

Random Dynamical Systems
Rua Murray

Random dynamical systems are obtained by a coupling a known family of deterministic systems to a driving stochastic process. The resulting behaviour is one kind of "non-autonomous dynamics". For deterministic dynamics, questions as simple as "how do you define an attractor?" have fairly straight-forward and unambiguous answers. However, in the random setting, the answers are more complicated and interesting. This project will investigate ergodic properties of random dynamical systems, focussing particularly on what insights "random transfer operators" can give about the behaviour of random dynamical systems. Some numerical calculation in Matlab (or similar) will be required. Applications include mass transfer/mixing in nonlinear fluid flows.

Synchronisation and Coupled Dynamical Systems
Rua Murray

This project will begin with a review of the well-studied and well understood topic of synchronisation in dynamical systems. The basic setup is that two or more identical systems are allowed to run from different initial conditions, but a small amount of coupling is present between them. Even when the dynamics is chaotic, orbits of the various oscillators can converge to a synchronised state. After learning the mathematics behind this phenomena, the project could proceed in any of several directions: studying chaotic synchronisation to encrypt communications; investigating the effects of multiple-timescales on synchronisation; exploring "approximate synchronisation" between similar, but non-identical systems. Applications include oscillatory behaviour and signal propagation in (excitable) biological tissues.

Linear Programming
Chris Price

This project looks at applications of discrete linear programming and solution methods via linear programming and other relaxations. Math303 is required as a prerequisite.

Computational Optimization and Applications
Rachael Tappenden

Optimization plays a crucial role in many modern, real-world applications, from finance to medical imaging, and from scheduling problems to big-data. Below is a list of several projects that students may be interested in working on.

- Portfolio Optimization: Given some initial budget, and a basket of financial instruments (stocks, bonds, etc), how should one allocate their funds in order to maximize their expected profit? Two questions that one may wish to answer are: (i) What kinds of algorithms can be used to efficiently solve such problems, and (ii) There are many ways of modeling portfolio optimization problems, but what is 'best'?

- Medical Imaging and Optimization: For many imaging modalities, including Computed Tomography (CT) and Magnetic Resonance Imaging (MRI), one must solve an 'inverse problem' to obtain an image. In this project we will investigate CT image reconstruction. We will see that the CT 'inverse problem' often involves the solution of a large linear system of equations, and the student will investigate algorithms for solving such systems.

- There are many other projects that may be of interest to students. General topics include: solving large scale linear systems efficiently, algorithms for determining the eigenvalues of large, sparse matrices, and algorithms for big-data problems. See me for further details.

For these projects, it would be helpful if the student had some knowledge of Matlab and of applied linear/matrix algebra.

Computing the Infinite
Hannes Diener

Since most mathematical objects are ideal/infinite, but computers can only ever deal with discrete/finite objects, it seems that we almost always have to confine ourselves to using approximations and shadow types (such as 'float' instead of the reals, or 'int' instead of the natural numbers) when transferring mathematical ideas to a computer.

Surprisingly though, it is possible to compute with (a large class of) infinite subsets of natural numbers, real numbers, and so on, not just as approximations, but as the actual objects. In this project a student should investigate some of the mathematical background of such an approach, and ideally implement some of the ideas. Depending on the student's background the project can either focus on the theoretical mathematical foundations, or on the practical implementation. For the first a background in logic is welcomed (for example MATH130, 230, or 336), for the second some familiarity with a programming language which allows elements of 'functional programming' (such as Python, Haskell, C++) is useful.

Philosophy and Foundations of Mathematics
Maarten McKubre-Jordens

What is mathematics? What is a mathematical theory? What are the objects of mathematical thought? Why is mathematics, an abstract subject, so effective in the real world? And when a mathematician proves something, what are they actually doing? In this project you will study developments in the philosophy and foundations of mathematics. Depending on your interest, the project may investigate philosophical positions concerning mathematics, mathematical truth and provability, or the limits of mathematics.

Non-classical Mathematics
Maarten McKubre-Jordens

Why are some mathematical questions not just unanswered, but unanswerable? And what is so bad about contradictory theories? The last few decades have seen a burgeoning in non-classical logics. In this project, you will investigate the relationship of these logics to mathematics. You will discover that there are many more facets to mathematics than many - even practicing mathematicians - realise. Moreover, you will learn some new mathematics non-classically, and gain insight into distinctions that cut mathematics at its very joints: the limits of provability, or the meaning of mathematical statements. Depending on your interest, the project may take you into: the emergence of logics from mathematics; the application of logics to mathematics; constructive mathematics; or paraconsistent mathematics.

History of Mathematics
John Hannah

The second half of the 19th century and the first half of the 20th century saw the introduction and development of several quite abstract branches of mathematics. Examples include abstract algebra (groups, fields, rings, ideals, modules), functional analysis, topology, and category theory. In this project you will follow this development in a topic of your own choosing, and in the process you will both learn some new mathematics, and try to discern the influences (mathematical, philosophical or cultural in origin) which led to these developments.

Ecological Networks
Alex James

From the Amazon rainforest to the Okeover stream species interact with each other in complex systems. These interactions come in many forms: competitive - they both compete for the same resource, predator-prey - eat or be eaten, mutualistic - both benefit from the relationship. How does the architecture of these interactions affect the behaviour of the system? Should a species interact with as many others as possible? Should they form groups or cliques? Is there a conflict between good for an individual and good for the system?

Useful pre-requisites for this project could include dynamical systems, discrete maths and some Matlab or other computer programming. An interest in ecology is also helpful.

I also have other projects available that combine maths and ecology. Please contact me to find out more.

Goanna microhabitat usage
Alex James & Daniel Gerhard

Australian Goanna are subject to a range of predators (mostly introduced!). In this project you will use new data collected by researchers at Landcare Research to understand how Goanna respond to predators in different habitats.

This project will give you an opportunity to do novel research on real ecological data. The project will have a co-supervisor from Landcare Research giving you a chance to make important contacts there and learn how data analysis is done outside a university environment.

Modelling Collective Cell Movement
Mike Plank

Collective cell behaviour is the driving force behind many physiological processes, including embryonic development, tissue repair and tumour growth. Experiments on collective cell behaviour typically collect data at the level of the population rather than the individual cell. WeâĂŹd like to be able to translate data from observing populations of cells into knowledge about how individual cells work and how they interact with their neighbours. This project will approach this problem using approximate Bayesian computation (ABC). At its simplest, this involves sampling model parameters from a prior distribution and simulating cell behaviour. If the model output is âĂIJcloseâĂİ to the experimental data, the parameter values are accepted as part of the posterior distribution, otherwise they are rejected. This will be used to estimate quantities such as cell proliferation and movement rates and the strength of interactions with neighbouring cells.

This project will require some experience of a computer programming language, e.g. Matlab and an interest in working with biological data. Prior knowledge of Bayesian statistics is NOT required.

Smaller Fish to Fry?
Mike Plank

Modern fisheries management is almost universally based on the principle of protecting small fish from capture and targeting large fish. Fisheries that do no conform to this dogma, such as small-scale African fisheries, are seen as destructive. But mathematical modelling and empirical experience has shown that, actually, these fisheries can be more sustainable than you might expect. The aim of this project is to investigate the optimal size of fish to catch from the point of view of an individual fisher. This is a game theoretical problem, because the choices made by one fisher can affect the fish population, and therefore the outcomes for other fishers. Of primary interest is the question: if each individual fisher behaves so as to maximise his/her own yield, what does the aggregate fishing pattern look like across the whole fishery? Are regulations needed to stop the fishers driving the fish population to extinction and/or driving each other to economic penury? Or can a self-organising group of individual fishers be ecologically and economically sustainable?

This project will require some experience of a computer programming language, e.g. Matlab, and an interest in using mathematical models to investigate problems in ecology, sociology or economics.

Tree-Child Networks
Charles Semple

Phylogenetic networks generalise phylogenetic (evolutionary) trees by allowing for non-tree-like evolutionary events such as lateral gene transfer and recombination. The mathematical study of phylogenetic networks is recent and, arguably, no more more that fifteen years old. Amongst the many subclasses of phylogenetic networks being studied, tree-child networks are emerging as an increasingly important class. What makes them so important? How do they relate to other classes? In this project, we investigate these questions. There are no prerequisites for the project.

Negative Correlation
Charles Semple

It follows from the work of Kirchhoff (1847) that the spanning trees of a connected graph G are negatively correlated. That is, for edges e and f of G, it is more likely that a spanning tree chosen at random contains e than one that contains e knowing that it also contains f. This seems intuitively clear but, nevertheless, still requires proof. What if, instead of choosing a spanning tree, we choose a forest at random? Does the analogous result still hold? The purpose of this project is to explore this and related problems. While some knowledge of graph theory would be helpful, it is not a prerequisite for the project.

The Banach-Tarski paradox and other paradoxical constructions
Gunter Steinke

Can you have your cake and eat it too? It seems quite simple: cut your cake in little pieces and reassemble them into two cakes each the size of the original one. Then you can eat one and keep the other. If this sounds too good to be true it is–for real life cakes that is. However, for mathematical cakes with all the intellectual nourishment they provide this can be done.

The project explores what is known as the Banach-Tarski Paradox: constructions in which sets in Euclidean space can be partitioned into subsets which, after certain transformations, can be rearranged to yield sets with very different properties. For example, the project looks at sets that when reassembled produce two copies of themselves and ways to reassemble a disk into a square of equal area.

One may also look at other constructions which are contrary to intuition. For example, one usually thinks of curves as 1-dimensional objects and something whose graphs can be drawn; they should be differentiable except for a few points. However, one can actually fill the entire unit square (a 2-dimensional object) as the continuous image of the unit interval. The project explores how such space-filling curves can be constructed and what properties they must have.

Regular tilings and crystallographic groups
Gunter Steinke

A tiling (or tessellation) of the plane is a covering by non-overlapping polygons like tiles in a parquet floor extended infinitely in all directions. The most symmetric and aesthetic tilings are the regular ones in which all polygons involved are congruent and regular. Similar configurations can be constructed on the sphere, which then correspond to the Platonic solids, and other surfaces. The symmetries of these tilings lead to groups which can be used to describe and characterise the tilings from which the groups arise. In case of Euclidean 3-space one obtains the so-called crystallographic groups, which describe the symmetry patterns of crystals, but the term crystallographic group is also used in case of patterns in the plane.

This project explores the concepts of symmetry groups and crystallographic groups investigates how they are used to obtain (and classify) all regular tilings of the plane and how to create some tilings on other surfaces.

The Mathematics of Natural Disasters
Phil Wilson & Miguel Moyers-Gonzales

Geologic processes can adversely impact human life and infrastructure, representing a significant risk to national security. For example, volcanic ash can choke emergency generators, pollute reservoirs, and damage hydroelectric turbines. This project will model selected aspects of natural disasters using mathematical and numerical techniques. The aim is to better understand the underlying physical processes in order to mitigate their consequent adverse effects. The project is part of a broader inter-disciplinary research group. The ideal candidate would have a good background in PDEs and MATLAB, but all applicants will be considered.

Projects in Statistics

Environmental Statistics
Jennifer Brown

Environmental monitoring is a fast moving, and important field of research. Data on environmental processes such as changes in water quality, endangered species distribution, weed invasion, and biodiversity are used to inform and guide how we manage our environment.

One use of environmental data is to build models to predict species distribution, and to predict the effect of environmental changes. In this project we will look at different methods used to collect field data and the effect of these differences on prediction models. We will use computer simulations to model data collection and analysis.

Bayesian Data Analysis
Elena Moltchanova

Bayesian statistics offers a different world view and often allows for richer and more flexible inference than the standard classical techniques.

1. One area where Bayesian approach is particularly useful is in jump processes where an unknown number of abrupt changes might have occured during the monitoring period. Reversible Jump Markov Chain Monte Carlo (RjMCMC) is one of the methods applicable in such situations. The project will concentrate on construction and implementation of this algorithm to a long-term multivariate time series. Good R programming skills and previous familiarity with Bayesian Inference (STAT314/STAT461) required. (No prior knowledge of time series analysis is necessary)

2. Estimation of parameters of stochastically truncated distributions is another area where Bayesian approach is useful. Instead of having a clear cut-off points, samples from stochastically truncated distributions include observations from the original non-truncated distribution with some probability. The estimation is thus concerned not only with the original distribution but also with the inclusion function. Reconstruction of historical distribution of population heights from the records available for army and navy recruits is one area of interest to economists, social scientists and anthropologists and forms the focus of this project. Good R programming skills and previous familiarity with Bayesian Inference (STAT314/STAT461) required.

Extreme Value Statistical Modelling
Carl Scarrott

The statistics you meet on undergraduate course typically focus on the capturing the "usual" characteristics of a process (e.g. properties of mean, median or variance). Extreme value statistics focus on understanding the unusual or rare events of a process. Extremes are of interests in all sort of different fields, for example:

1. financial risk (e.g. estimating Value at Risk for risk and portfolio management)
2. engineering (e.g. designing structures to withstand the stongest forces they could be exposed to, e.g. wind exposure on bridges or forces on buildings due to earthquakes!)
3. environment (e.g. are Christchurch's winter air pollution extremes reducing due to government interventions?)
4. IT (e.g. can our servers and network

cope with peak demand when your assignments and projects are due in?)

I have various honours projects in extreme value modelling for financial, environmental, medical and industrial applications. So if you are thinking of becoming a statistical "extremist" then get in touch.

Below are two projects which are accompanied with a $1000 stipend. Students with work with Dr Hamish Jamieson from the Otago Medical School/Canterbury District Health Board and an academic staff member from our School.

Canterbury District Health Board Dataset

PROJECT ONE
To assess the use BlueFern supercomputer at UC to analyse the a large dataset from the Canterbury District Health Board on patient health. There are many factors that can be explored and modelled, and all are useful for the CDHB, e.g., rest home admission, mortality, hospital admissions. The dataset is part of a nationwide, and international, programme to assess health in the ageing population.

PROJECT TWO (With the Brain Research Institute)
To use the Canterbury District Health Board dataset on health in the ageing population to identify the characteristics of people with Parkinson's Disease and possible predictors of the disease. The second part of this project will look at what confounding factors influence the outcomes in patients with cognitive impairment.

For more details, please contact Jennifer Brown.

University of Canterbury
Te Whāre Wananga o Waitaha
Private Bag 4800
Christchurch 8140
New Zealand

Telephone: +64 366 7001
Freephone in NZ: 0800 VARSITY (0800 827 748)
Facsimile: +64 3 364 2999
Email: info@canterbury.ac.nz
www.canterbury.ac.nz