Probability: the language of the people! ... the language of science ??

Frank Lad*

Abstract

The modern conception of scientific method as an skeptical and value-free learning procedure is the source of the distrust and disrespect shown to science by many people today. In defiance of this conception, this lecture proposes the crucial role of beliefs and values in the conduct of scientific activity. In this mode, science can be recognised as an important component of democratic human development which involves other modes of learning about Nature as well. The conception of science as a belief-centered and value-oriented process is supported technically by the operational subjective theory of probability, developed notably through the stimulus of the eminent Italian mathematician, Bruno de Finetti.

Key Words: scientific method, metaphysics, skepticism, gambling, rational belief, utility valuation, empiricism, subjective probability, hypothesis testing, spiritual experience.

1 Prelude

Despite an impressive array of achievements over the past twenty years and a staggering record over the past century, scientists today find themselves plagued by resounding popular doubts over the validity of their claims and the value of their results. The doubts are displayed in a variety of ways. In practical matters they show up in the increasing numbers of young parents who refuse to have their children innoculated against truly debilitating diseases such as polio and rheumatic fever, and in the growing healthy distrust of genetic modification of our edible plant heritage, which seems designed to increase the powers of the agro-business community over our food supply and farming culture. On more esoteric issues, popular doubts of the scientific imagination regarding the origin of the cosmos and the evolution of species are aired regularly in both secular and religious media. In English speaking countries, the most popular fiction books for early school age readers today form a series that focuses on a school in which children supposedly learn procedures for performing magic and casting spells. Of course fantasy has its place in imaginative thinking, but to those of us who place great value on analytic and rational constructions, the state of popular support for the development of science is worrisome, to say the least.

In my view, this situation has been provoked by the intransigent attitude of scientists themselves regarding the epistemological basis for their claims to truth and toward the so-called scientific method by which such claims are substantiated. It is important that we address honestly the reasons this situation has arisen, and challenge the misconceptions about the nature and value of scientific activity that have festered within our community.

I see two fundamental problems in the image of science that we commonly portray to the world. The first of them is a technical matter that will be the subject of most of my arguments in today's lecture. It concerns the subject of probability and the role it plays in science. The second of them concerns the limitations of the rational analytic method of

*Frank Lad is a consulting statistician and a research associate in the Department of Mathematics and Statistics, University of Canterbury, Christchurch, New Zealand/Aotearoa. This lecture has been prepared at the invitation of the Conference on Science and Democracy 2, Palazzo Serra di Cassano, Napoli, 12-14 June, 2003.

knowing, scientific method if you will, and what might be learned by scientists from other modes of knowing. I shall speak to this issue in the conclusion of my talk.

In the main body of this talk I would like to dwell on the substance of the scientific method, focusing on a tremendous misconception that scientists have foisted upon the public. This has yielded problems in the extent to which our valuable insights are recognised by non-scientists, and has also provided problems within the techniques of science itself. I shall begin with a brief sketch of the image of science that has developed from its European renaissance through its objective and skeptical self-understanding by the end of the millenium. The mainstream scientific understanding of probability and statistical inference shall be highlighted. I shall then challenge this framework with some troublesome questions, some recognition of the relevance of popular interest in gambling, and a presentation of the contemporary understanding of subjective probability as understood by an active minority of probabilists and statisticians. The mathematical formulation of this viewpoint supports a common sense alternative to the value-free and skeptic understanding of science, emphasising instead the central role of intuition, belief and values in the conduct of our scientific activity. I shall then outline briefly three particular aspects of statistical practice that exhibit the need for a new understanding. After a summary statement of the position I propose to you, I will conclude with some comment on ways of knowing that lie outside the scope of science, and their relevance to matters that concern us all.

2 The post-renaissance doctrine of scientific method

Inspired both by the new possibilities of astronomical observation allowed by the telescope and by the extension of mechanical engineering techniques, renaissance science bequeathed to subsequent understanding the image of Nature as a machine. In the early heady days of promise, the machine was imagined to be so precise and regular as to generate the heavenly "music of the spheres" as well as even the natural history of the "Tiger, tiger burning bright."

Tiger, tiger burning bright
in the forests of the night
what immortal hand or eye
could frame thy fearful symmetry?
...
When the stars drew down their spears
and watered heaven with their tears,
did he smile his work to see?
Did he who made the lamb make thee?
.
William Blake, 1794, *Songs of Experience: the Tiger, st. 1, 5*

However, irregularities of measurement were soon recognised as problematic to scientific understanding the mechanism of the machine already in the eighteenth and nineteenth centuries. Moreover, the intensive atomic experimentation of the early twentieth century led to the specification of a supposed "randomness" as an intrinsic feature of the generating machine. Estimating procedures of "statistical mechanics" came to be seen as central to scientific inference, invading the imagination of theoreticians in virtually every field of physical and social science.

3 The emergence of objective probability

The understanding of Nature as a random generating device with stochastic laws that can be tamed by attention to an impersonal, skeptic and objective statistical method became virtually universal by the mid-twentieth century. See Hacking (1990). Central to this spectre of science was the imperative that scientists behave in a way that denies any personal interest in the outcome of their experimentation and inference, and that precludes any influence of their own beliefs about their subject matter on their work. Access to the realms of probability was explicitly not allowed to the common person with mundane interests such as how long and at what temperature to reheat a dish of two-day-old lasagne before eating it. Probabilities were deemed relevant only to special types of events that are repeatable, at least in principle, and that exhibit a stable frequency of occurrence among the repeated instances of their setup. This attitude is virtually universal among the texts on so-called "objective probability," but a classic reference addressing this topic would be the oft-reprinted text of Gnedenko (1968). It was explicitly argued that probabilities are not relevant to any old problem about which people are uncertain, and that only ordained statisticians are empowered to make assessments of its relevance. The prevalence of this view is discussed in the critical and imaginative writings by Nalimov (1989) which should be more widely known.

4 Questions about this outlook

Of course the common person has been kept at arms length from a cherished "full understanding" of the secrets of probability by specialist language replete with important sounding incantations such as "homoscedastic stationary stochastic processes." However, naive questions continue to ring in the ears of specialists, much as the haunting cries of the child in the ears of the emperor who wore no clothes – "These probabilities ... you cannot see them, hear them, touch them, taste them or smell them? How are we to know anything at all about them? And what do they have to do with the process of science, which I thought is meant to be founded on empirical standards of proof?"

The only response to such questions that is given by the high priests of mainstream contemporary statistical practice is that "Oh, unfortunately you are not able to understand," presumably because the questioner is not conversant with the language of measure spaces in which the formal axioms of probability have been couched.

Throughout the world today, in every country and in every language in which courses in higher education are conducted, training in statistical methods is an essential requirement. However, I feel confident in asserting that if every person in the world who has achieved a baccalaureate degree or beyond would be asked "which subject did you find the most frustrating and detestable of all the courses that you were required to study in preparation for your degree?" the almost universal answer would be "Statistics!" As a statistician who has studied the foundations of probability for more than thirty years, I believe there is good reason that this situation has emerged. When faced with real substantive questions that undermine the credibility of the computational procedures they promote, the teacher is inevitably left to rely on the power of his/her personal authority as teacher to resolve them. The student is required to submit to the authority to achieve the desired degree, but not without incredible resentment. The reason that authority is the only support for the programme is that at its foundation, the modern mathematical theory of probability and statistics is based on shoddy thinking and errors. But I am perhaps getting a bit ahead of myself.

5 Popular interest in gambling and its relevance

People today are very aware that the times we live in are quite different in many ways from every era before us. As a result, each of us is uncertain about virtually every empirical question that lies beyond our immediate gaze at any time. Outside of our high-brow institutions of learning, "the people" have developed a time-honoured way to resolve their disputes about empirical questions that interest them. The procedure is simple – "Would you like to bet on that?" Today, personal betting for amusement amounts to a multi-billion dollar industry in most regions of the world. Both American and European football attracts phenomenal interest and monetary commitments every week. In India, the subject of the betting is more commonly cricket, but the betting behaviour is equally avid. With such amounts of money at stake, it is not at all surprising that we have recently witnessed the international scandal of high profile players cheating at the game by limiting their efforts at the bequest of monied gamblers. Among Chinese people too, the attraction to gaming is legion, as will be learned by anyone with eyes to see at the haunts of licensed gaming facilities around the world.

I am not here to applaud the common fascination with gambling, but merely to recognise it as a widely understood feature of denominating human uncertainty in numerically graded units. You can bet on anything that has an observable outcome through sight, sound, taste, smell or touch. This common insight is in fact the basis for the reemergence of the subjective theory of probability that I shall now address.

6 Probability, the logic of uncertainty

It may come as a surprise, but the structure of rational betting schemes has provided a basis for the development of the formal mathematics of probability, providing an alternative to the bogus scientific method that requires incredulity and non-valuation by its practitioners. In fact, there has been an active tradition that has supported this viewpoint in mathematics and science throughout the renaissance, with eminent promoters such as Laplace and De Morgan. However, it has been aggressively stifled at least in the era since 1850. The real modern leader of the subjective understanding of probability as the logic of uncertain knowledge was an amazing Italian mathematician, Bruno de Finetti, who lived from 1906-1986. He published his early research results before the second world war, and completed his long professional working career at the Assicurazione Nazionale in Trieste, and as a professor at the University of Rome.

DeFinetti's great view was that the logic of certainty, so-called 2-valued logic, really has very limited applicability to scientific or secular matters because there is really so little that we are certain about. The logic of certainty applies to the relations between the various sentences about which we can affirm that they are true or that they are false. If you assert that A is true, and you assert that A implies B is true, then you must also assert that B is true as well, or else you will be contradicting yourself. However, in science as in life, there are very few precise empirical sentences about which we are certain. The scale of units of our uncertainties may range from the give and take of one billion years on matters of cosmology to the give and take of a pico-second on matters of computational accuracy, but the logical foundation for our uncertain knowledge is the same in all matters, no matter how esoteric or how mundane. About most empirical matters we are uncertain. Thus, it is imperative that we have a formal logic that constrains the numerical measures of our uncertainties about various sentences when they are expressed not as 0's and 1's (certainty), but by the numbers in between (probabilities). The mathematics of probability is the logic of uncertainty. It was the genius of de Finetti that constructed the formalities of this mathematics and its computable numerical applications.

Personal probability assertions are evaluated in terms of the prices that one is willing to pay for resolvable bets on observable measurements. The logical restrictions on an array of assertions are generated by the principle of coherency. It is specified by this requirement that the array of prices one is willing to pay for a sequence of bets does not result that the proponent becomes a net loser in the transactions under every possible scenario of how the bets might eventually resolve. The principle of coherency and the logic of uncertainty is a many valued extension of the principle of non-contradiction in the two valued logic of certainty.

The motivations behind de Finetti's constructions were populist and straightforward. He was insistent that the method of science for making observation and inference were no different than the rational methods available to the general person on the street who wanted to heat properly the leftover lasagne from two days before. No hocus pocus is required. Nor is a white suit.

On this matter he wrote one amusing story (see Gani, 1982) when he decided to ask the proprietor of a cafe what were meant by the odds for the week's football matches that were posted on the window of the shop, just to ensure for himself that his ideas about probability as conforming to the understanding of the general person on the street were correct.

The man at the bar looked at him quizzically and responded "Why those are the probabilities! You must be the only person in Rome who does not know what probability is!" The joke is that de Finetti was one of the few people in the world during the twentieth century who *did* know what probability is, not succumbing to the pseudo-scientific aberration of proposed fundamental probabilistic units of matter that are inaccessible to human sensation.

7 Statistical method: past, present and future

This is not the place to present a quick course on the operational statistical procedures that stem from the mathematical derivation of the subjectivist viewpoint. I have explained that in great detail in my text *Operational Subjective Statistical Methods: a mathematical, philosophical and historical introduction* (Lad, 1996) which may interest you. I have also recently published a pedagogical paper for the Asti Summer School on Applied Statistics, presenting the details of an application of this method. It addresses specifically the zoological problem of understanding whether the power of the tides (and thus the phases of the moon) have any relevance to the phenomena of strandings of herds of whales on ocean beaches (Lad, 2003). At the moment I shall merely say a few general words, and then focus in a little more detail on three particular problems with current mainstream statistical methods.

Objectivist methods presume that Natural history is generated by a stochastic mechanical process whereby events are determined randomly according to unknown and unobservable probabilistic structures. The outlook proposes that scientists can identify the appropriate family of possible generating structures for phenomena in various applications. The methods are then oriented toward determining via statistical testing whether the unobservable generating parameters of these structures fall into specified numerical ranges. Students of applied sciences who have been forced to learn a catalogue of t-tests, F-tests, and Chi-square tests will know too well the delights of such a viewpoint.

To some extent, the subjectivist has really nothing to say to the promoters of such procedures, for we deny at the very beginning any metaphysical belief in such unobservable stochastic generating structures. We recognise that science is based on numerical recordings of empirical measurements of our sense experiences. In any experimental or exploratory investigation, scientists are typically uncertain about what these measurements might be. Based on a mathematical characterisation of their uncertainties about a sequence of such measurements, subjectivist statistical theory provides computational procedures for determining how to change their uncertain assessments about subsequent experiments based on the observations they have made in newly acquired data. What are real are our experiences and their numerically measured summaries. The probabilities merely represent our changing uncertainties about the experiences we shall have in the future.

Scientists' ability to make use of these procedures requires that they squarely admit their uncertain beliefs, attempt to formalise them (perhaps with the help of a consulting statistician), and take responsibility for them. They can then honestly present the method and the results of this activity to their colleagues in a form that encourages others to change their beliefs about Nature in the same way. There is no recourse to an objective statistical method that forces others to make the same conclusions and that allows the inferring scientist to disclaim responsibility for the inferences he/she promotes. This procedure allows for recognisably valid controversy in science. It also requires that scientists admit to the public that their inferences are based on their assessments of their uncertain beliefs, rather than hiding behind their white coats of non-responsible "objectivity." There need not be any shame in this. This is in fact the scientists' glory, that they are able to concentrate their minds to an extent that they can formulate coherent, intriguing and fruitful beliefs regarding the Nature about us. I believe that the masses' willingness to applaud their achievements will grow along with scientists' willingness to admit their own humanity ... their uncertainty.

There is a theorem in probability, called Bayes' theorem, which codifies how inferential learning from new information is to take place in a coherent way. Because it has widespread applicability, the general body of statistical developments that stem from it is called "Bayesian Statistics." By and large, I find myself mildly supportive of the movement that goes by that name. I am only somewhat wary of the extent to which its proponents have not completely thought out the issues involved. Many of them still work in the vein that admits that unobservable probabilities generate Nature, but think that we are uncertain about what the values of those probabilities are. Thus they think in terms of subjective probability assertions regarding the values of unobservable objective probabilities. I hope you can share my suspicion of such a viewpoint as you look into these matters for yourself.

Without getting deeper into any such detail, I would like to discuss three specific examples of contemporary statistical method which the subjectivist perspective has addressed, pointing out how the old fashioned methods of "objective probability and statistics" really have misled us in producing the results they propose to support. I will be brief, but hopefully informative.

7.1 Hypothesis testing and "significant" scientific evidence

Most of us would be familiar with standard claims regarding the "statistical significance" of data with respect to a particular test. According to commonly used "objective" statistical methods, it is typically regarded as powerful evidence against a null hypothesis if it can be rejected at a very small significance level based upon a large number of observations. Let me refer you to a very striking analytical result that lets us know that such "significant data" do not necessarily amount to evidence in favour of rejecting the specified null hypothesis at all!

The result is called the Jeffreys-Lindley paradox, named after two English statisticians who generated it midway in the past century (Lindley, 1957). Suppose you decide to reject a null hypothesis in favour of an alternative once you are able to reject it at some specific significance level, as small as you like. (Some journals which support such a decision procedure print a star next to parameter estimates that are significantly different from zero at the .05 level, two stars if they are significantly different from zero at the .01 level, and three stars if at the .001 level!) Under very mild assumptions regarding prior expectations, it can be shown that the posterior probability in favour of the null hypothesis being true, conditional on data that leads you to reject the hypothesis at this desired significance level, becomes arbitrarily close to 1 as the number of observations involved increases. This is and should be really a shocking result to anyone who is aware of the ubiquitous use of so-called significance testing in medicine and engineering. It was one of the motivating factors for the technical development of subjectivist Bayesian procedures in the past half century. But these have been regularly rebuffed by academic scientific claims to "objectivity" that have mistakenly but proudly denied the role of beliefs and values in the conduct of science.

Serious attention to the foundation for the now too common practice of hypothesis testing will show that there are two types of possible error that characterise any statistical decision – the one of them viewed as rejecting a hypothesis when it is true (called type I error), and the other that of accepting a hypothesis when it is false (called a type II error). However, the common practice of significance testing today typically ignores the relevance of type II errors and their probabilities, because these are very difficult to compute, if not impossible, according to standard conceptions. The Jeffreys-Lindley paradox revolves upon this fact. Without careful consideration of the entire array of possibilities, the fact that some observed data are unlikely when the null hypothesis is true can disguise the fact that they are even more unlikely when the null hypothesis is false! Further research on this type of result has been developed under the provocative name of procedures for "testing to a foregone conclusion." See for example, Cornfield (1970, pp. 18-22).

7.2 Misconceptions of the extent of our uncertainties

A second area in which we can make very general technical statements about the inadequacy and the misleading nature of statistical results based on standard procedure concerns the variance stated for public statistical forecasts. Standard procedures are based on estimated values of the parameters of statistical generating models. The estimations are usually based on some form of the maximum likelihood method. Once the point estimations of the parameters are computed, forecasts for subsequent measurements are pronounced as those calculated from the generating model with this specific parameter configuration. What is suppressed in this method is the recognition that even in objectivist terms, we are still uncertain about the generating parameter values themselves, so there is a further variance component that should be added into the published forecast variance, but is not. Thus, quite generally, across all subject matters of application, scientific forecasts based on the standard statistical methods of the twentieth century understate the variances of forecasts. This is recognised empirically from time to time when "improved estimates" are made that are far away from the so-called "confidence intervals" of previously announced scientific knowledge. This has occured even in the realms of such central concepts as "the fundamental constants of physics."

The advent of Bayesian methods has done much to highlight this inadequacy in standard statistical methods. Bayesian methods increase the stated uncertainty variance because they naturally average their forecasts over the forecasts based on all possible model parameters. It should be noted that objectivist statistical theory resists such model averaging on the very philosophical grounds that I am challenging in this talk. For objectivists reject the validity of scientists' assertions of uncertain opinion about model parameters as relevant to their supposed scientific method.

7.3 Assertions of value – the yardstick of uncertainty

I have claimed in this talk that the proper formulation of science requires that we admit and even proclaim scientists' beliefs and values as central to the conduct of valid science. Yet up to this point I have concentrated my discussion on the profession of scientific beliefs, denominated in units of subjective probabilities. I would like now to remark briefly about the role of value assertions.

The supposed glory of standard statistical procedures is that they can determine the "significance" of an experimental result seemingly without anyone asserting beliefs or values for which they are personally responsible, and seemingly without any valid retort from non-scientists who live in a world of beliefs and values. On the one hand, I would like to proclaim that this popular viewpoint does not really make sense even in the foundational arguments that provide the basis for the objectivist procedures developed in the past century. As I had mentioned earlier, there are two types of possible error that characterise any statistical decision – the one of them viewed as rejecting a hypothesis when it is true, and the other that of accepting a hypothesis when it is false. In the very earliest days of the formulation of this way of thinking it was recognised and stated explicitly that the level (of type I error) at which it is appropriate to conduct a statistical test necessarily depends upon the prior believability of the hypotheses under consideration, and also upon the relative valuation of the losses one would incur in making the two types of errors. (See for example the famous statement of Lehmann, 1959, pp. 61-62.) However such protestations have by now been long forgotten by the sales arms of "scientific statistical software" companies who like to proclaim to non-mathematical users of statistics that their wares will "compute the significance you require for you!"

The operational subjective characterisation of science is much more explicit about the importance of assessing one's value judgements, and even specifying them numerically in terms of a utility function. Just as probabilities are personal assertions of uncertainty, which can be different for everyone, this is also true for utilities which are denominated units of value. Differing probability assessments can be brought closer by experimental designs that take into account the sources of argumentative differences of opinions. But values are matters of taste that differ among people, and can only be brought closer by methods of persuasion and conversion or negotiation. Many of the most important technological innovations today are controversial not only because of differing opinions about their practicality and their safety but because of differing valuations of the returns they will bring and of the consequences of possible mistakes and accidents. Let me raise three areas specifically for consideration.

7.3.1 The value of nuclear power production

The continued expansion of nuclear power generation is a worry to many people of the world. Not only is there concern about the consequences of systems that run out of control, such as we have experienced at Chernobyl and Three Mile Island, but there is a seemingly irresolvable issue about what to do with the radioactive waste material that must be buried somewhere to neutralise its harmful environmental effects. Most everyone is happy to use the power, but no one really wants the waste. There is no "objective" scientific resolution possible of these issues. I believe that every person has the right to his and her own judgement of the relative values of the goods and the bads involved. Surely scientists have no special claims to the validity of their value judgements *per se*. But when they do argue in favour of one proposal or another it is crucial that they explicitly state their relative values for the possible outcomes of the generation process.

Dumping of chemical and nuclear wastes on remote Pacific islands (remote at least from Europe and the USA), for example, can only be supported rationally by a value system that explicitly demeans the value of unpoisoned life of people, animals and plants who live there. What is most disconcerting to those of us who are interested by issues of science and democracy is that the people of the nations upon whom the nuclear waste is foisted through political deals and the wining and dining of politicians are not the ones who use the power the nuclear facilities produce.

7.3.2 The value of exploratory science

As human life and its designs encroach deeper and deeper into the entire space of the planet earth, it is time that we step back and think about the relative values of human life and the complex of all other living organisms. It is not necessarily true that human desires should be satisfied at the expense of every other living organism. Of course a judgement in favour of this valuation can be made as a matter of taste, but it should be made out in the open. I do not agree with such a judgement as a basic principle. In my scheme, each such question must be addressed specifically on its own merits. Even the value of scientific investigation itself can be sometimes subservient to the value of allowing forms of life to continue unimpeded by human intrusion. I would assert that wild animals and plants have value and even rights too, even though they cannot vote in any democratic process. I am not alone in such a valuation. See, for example, the extensive and controversial writings of Peter Singer (1990). The rights of exploratory sciences to investigate every crevice of existence are not unassailable. It is important that we make public any programs to extend scientific incursions into the remaining environs of Nature, and to debate their value. In deciding such matters, the desires of scientists need not necessarily be satisfied in opposition to alternative valuation schemes.

7.3.3 The value of human life

The self importance of humanity must especially be challenged in the field of medical technology and intervention. I would like to challenge publicly the very widespread conception that the prolongation of human life, *per se*, is always the highest value to be furthered whenever possible. Quite the reverse, I believe the scientific community specifically, and the members of the high-consumption societies in general, have much to learn from pre-scientific societies about the value of completing one's life in harmony with the powers of the living whole. I believe that learning to experience death with dignity by peoples who have forgotten or suppressed this possibility would be an important development of the human spirit. It also brings to mind the importance of our questioning the scientific development of reproductive technology that defies the limitation in the fertility of human beings. Of course the coming to grips with the emotional experience of the inability to conceive children is not a simple task, but just because there are people who are willing to pay enough to surmount this impasse does not make it right that research dollars are spent on such problems to the extent that they are. At our current numeration of some 6 billion human beings and counting, it is well worth our re-evaluation of the importance of human life in the context of our re-evaluation of the value of the rest of Nature in which we live.

I have chosen to raise these issues of human valuation, perhaps out of the blue and out of the context of any particular human decision, because it is important that we recognise these are contestable issues. In pursuing scientific activity we must assess honestly our valuations of such questions and have the integrity to hold back our blind pursuit of the much touted "progress."

8 Summary

Let me summarise the substance of my argument up to this point.

The subjectivist statistical outlook challenges the current mainstream images of science as misleading at best. In that spectre, Nature is supposed to be a machine generating random outcomes through its atomic units, held together by stochastic principles of the laws of large numbers; and scientists gain access to it by transforming themselves into non-persons by denying any values or beliefs they hold regarding the empirical world they investigate. The alternative is completely formed and explicit. A progressive understanding of science is to recognise Nature as a living organism, not as a machine at all. As a living organism it has a beginning (the so-called big-bang), a lifetime in which it develops, grows, transforms and decays (natural history) and an end (the burnout of a star). Scientists, who are participants in this Nature, are individually and collectively uncertain to a greater or lesser extent about virtually every aspect of its course. Probability, and the statistical method of inference that derives from its structure, is a representation of a scientist's uncertainty (and scientists' uncertainties, when they agree) about the course of the history rather than a feature of the history itself. The goal of scientific method is to face calmly our uncertainty about natural history by expressing it logically in terms of probabilities. This formalised uncertainty allows us to make sensible judgements about how to design appropriate experiments and historical investigations to reduce the uncertainty, increasing our understanding and appreciation of Nature as we do. Moreover, when scientific uncertainty is formulated in this way, it can be used sensibly when people need to make democratic decisions by balancing our valuations of the uncertain outcomes of acts we may decide to engage.

The divergence of this new contemporary understanding of Nature and of science from the stochastic epicycles of the received imagination of Nature as a machine is a liberating achievement of the past fifty years of struggle in the foundations of statistics. It is time for the dissemination of the method and the imagination to become widespread and commonplace in the understanding of applied scientists who are not necessarily expert in the language of mathematics. It would be particularly appropriate for this realisation to gain acceptance in Italy, where students should learn about and be especially proud of the great methodological achievements of their compatriot, Bruno de Finetti. Through countless regular publications over the past century, he cheerfully and laboriously worked against the tide of international opinion in developing the most important aspects of an honestly human and useful subjectivist statistical outlook and method.

9 Finale

Having focused on the concept of probability and the role it ought play in our reconception of science, I should like to conclude in another vein, outside the scope of scientific discussion. I believe that the future of healthy scientific contributions to matters of social and technological concern to society rests on a vigorous public recognition that the rational analytic methods of scientific progress do *not* constitute the *only* valid and useful way of learning about the world and about ourselves. The methods of science, in all its subject areas from physics, chemistry and geology to biology, anthropology, economics, sociology and psychology, and on to astronomy and cosmology, are based on studied uncertain inferences that we make from empirical observations about further empirical observations. These are conducted in a formal language that allows extreme precision in the content of our communications. However the practice of the method, which requires that we assert our beliefs via probabilities and avow our values via utilities, severely limits the breadth of Nature that we can experience in this way.

There are surely further modes of knowing that defy contact and expression in this mode. These experiences are best engendered not by conscious controlled analysis, but rather by the formless merging of our beings with the larger being of life. Just because we cannot speak precisely about the content of such "knowing" in the technical language of science does not negate the validity and value of such experience. This has been recognised by wise people in every society and time. Listen to the words of Lao Tzu of ancient China who spoke in the *Tao Te Ching* (I.3.) of a way of knowing requiring that

> Always without desire we must be found
> if its deep mystery we would sound.
> But if desire within us be
> its outer fringe is all that we would see.

Alternatively, we could hear the same injunction in the different words of the Austrian Franz Kafka in his *Reflections on the Great Wall of China*:

> You do not need to leave your room. Remain sitting at your table and listen.
> Do not even listen, simply wait. Do not even wait, be quite still and solitary.
> The world will freely offer itself to you to be unmasked. It has no choice.
> It will roll in ecstacy at your feet.

I am referring here to the tradition that we commonly think of as spiritual experience. Just as the promoters of science have broadcast a misleading conception of its process and method, so too have the provocateurs of religious institutions. We have been led to think that science is a procedure which ignores value and shuns belief while religion thrives on human valuation and belief in the non-empirical. To conclude this talk I would like you to consider reversing your conceptions of these matters. Valuation and belief are crucial to the conduct of science, while they are actually inimical to the development of spiritual understanding which defies human self-important valuation and judgement. The content of this latter understanding is hard to put into words, but in every culture we have tried. The one universal theme that runs through the sayings of every one who honestly speaks of it is the simple song that "love makes the world go round." While non-analytic or specific, this message does translate into political action in many arenas. For example it leads us to defy short-sighted methods of agriculture that would make all of Nature subservient to human desires, that would annihilate all plant-eating insects and would prohibit people from entering the private mechanical plantation farms of the monocultured agrobusiness.

My warning to the promoters of "science" and its investors is that they not attempt to proclaim our analytic methods as the unique way of knowing the world. The respect and valuation of the results of our efforts from our non-scientific brothers and sisters will not be achieved by denying they have access to knowledge and valuation by methods of their own.

Rather than make further claims about "other modes of knowing", let me merely affirm them for myself, and recall the words of one who spoke of them in the very technical philosophical form of his Tractatus, "Whereof we cannot speak, thereof we must remain silent." (Wittgenstein, 1922, 7, p. 189) His conclusion at this time was a bit harsh. People can speak about such things, but not using logico-scientific language. It is the language of art and the heart.

10 Conclusion

This lecture has been entitled "Probability: the language of the people! ... the language of science ??" People of the earth everywhere are comfortable with the idea that they are uncertain. Although they may not be fully knowledgeable about the mathematical laws of its coherent assertion, codified in the theory of probability, and although they may make mistakes in its application, they are aware of its relevance to their knowledge of virtually every aspect of their lives. Thus, they are rightfully indignant at the attempts of Science to promote itself as having achieved somehow a method that escapes this common mode of knowing. Until scientists as a culture come to grips with the reality of uncertainty and value assertions in their own activities, they will continue to misconstrue the real achievements of their efforts and bear the brunt of public reluctance to applaud their results. The mathematically formulated theory of subjective probability provides a method for incorporating this recognition into our assessment of the validity of our own inferential work and our presentation of our achievements to the public. Today is the day to accept it as the rightful language of science.

11 References

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