

APPROXIMATION AND SPANNING IN THE HARDY SPACE, BY AFFINE SYSTEMS

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ABSTRACT. We find weak conditions on $\psi \in L^1(\mathbb{R}^d)$ with $\widehat{\psi}(0) = 1$ such that every function in the Hardy space is a linear combination of translates and dilates of ψ . More precisely, we prove for each $f \in H^1(\mathbb{R}^d)$ the scale averaged approximation formula

$$f(x) = \lim_{J \rightarrow \infty} \frac{1}{J} \sum_{j=1}^J \sum_{k \in \mathbb{Z}^d} c_{j,k} \psi(a_j x - k) \quad \text{in } H^1(\mathbb{R}^d),$$

where $\{a_j\}$ is an arbitrary lacunary sequence (such as $a_j = 2^j$) and the coefficients $c_{j,k}$ are local averages of f . This holds in particular if ψ is Schwartz class, or if $\psi \in L^p$ (for some $1 < p < \infty$) has compact support. A corollary is a new affine decomposition of H^1 in terms of differences of ψ .