Extreme Selections and Dimensions

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For a Hausdorff space $X$, let $\mathcal{F}(X)$ be the set of all nonempty closed subsets of $X$. Usually, we endow $\mathcal{F}(X)$ with the Vietoris topology $\tau_V$, and call it the Vietoris hyperspace of $X$. Recall that $\tau_V$ is generated by all collections of the form

$$\langle \mathcal{V} \rangle = \left\{ S \in \mathcal{F}(X) : S \subset \bigcup \mathcal{V} \text{ and } S \cap V \neq \emptyset, \text{ whenever } V \in \mathcal{V} \right\},$$

where $\mathcal{V}$ runs over the finite families of open subsets of $X$.

Let $\mathcal{D}$ be a subspace of the hyperspace $(\mathcal{F}(X), \tau_V)$. A map $\sigma : \mathcal{D} \to X$ is a selection for $\mathcal{D}$ if $\sigma(S) \in S$ for every $S \in \mathcal{D}$. A selection $\sigma : \mathcal{D} \to X$ is continuous if it is continuous with respect to the relative Vietoris topology $\tau_V$ on $\mathcal{D}$.

In this talk we mainly concern dimensions of a space $X$ which has a continuous selection for $\mathcal{F}(X)$ or $\mathcal{F}_2(X) = \{ S \in \mathcal{F}(X) : |S| = 2 \}$. Especially we give an answer to the following Question(Open Problems in Topology II, No. 394).

**Question** Let $X$ be a space which has a continuous selection for $\mathcal{F}(X)$. Then is it true that $\text{ind}(X) \leq 1$?