

# Titles and Abstracts of the talks

Number Theory Down Under  
25-29 August 2025

## Muhammad Afifurrahman

**Title:** Moments of restricted divisor functions (I)

**Abstract:** We derive asymptotic formulae for the higher-power moments of restricted divisor functions. In order to establish our main results, we study more general systems of multiplicative Diophantine equations and derive asymptotic and O-results in these directions. Joint work with C. C. Corrigan.

## Gustav Kjaerbye Bagger

**Title:** Detecting primitive elements in finite fields

**Abstract:** Consider the problem of showing existence of primitive elements in a subset  $\mathcal{A} \subseteq \mathbb{F}_{q^r}$ . One can prove a sieve criterion for existence of such elements, dependent only on an estimate for the character sum  $\sum_{\gamma \in \mathcal{A}} \chi(\gamma)$ . I will present this general detection tool and demonstrate its utility by tackling a problem of Fernandes and Reis with  $\mathcal{A}$  avoiding affine hyperplanes. The talk is based on joint work with James Punch.

## Chiara Bellotti

**Title:** Recent progress on explicit zero-density estimates near unity

**Abstract:** In this talk, I will present explicit zero-density estimates  $N(\sigma, T)$  for the Riemann zeta-function  $\zeta(s)$ , with a particular focus on those that are most effective near the line  $\operatorname{Re}(s) = 1$ . Estimates of  $N(\sigma, T)$  usually take the form  $N(\sigma, T) \ll T^{A(\sigma)(1-\sigma)} \log^C T$ , where  $A(\sigma)$  is a function of  $\sigma$  which tends to 0 when  $\sigma$  approaches 1.

These estimates are central to many problems in analytic number theory, especially in controlling error terms in the prime counting function and related asymptotics. After a brief overview of classical results, I will discuss recent progress, including results from my own recent work, which provide improved explicit bounds when  $\sigma$  is sufficiently close to the 1-line. I will also briefly touch on existing estimates for Dirichlet  $L$ -functions and outline potential avenues for generalization. The talk is based on both published papers and work in progress.

## Subham Bhakta

**Title:** Quantum ergodicity for cat maps modulo prime powers

**Abstract:** Cat maps are hyperbolic automorphisms of the torus, with quantum analogs given by unitary operators on the Hilbert space  $L^2(\mathbb{Z}/N\mathbb{Z})$ . P. Kurlberg and Z. Rudnick (2001) showed that for almost all moduli  $N$ , the eigenfunctions of the quantum cat maps are uniformly distributed (quantum ergodicity). J. Bourgain (2005), and more recently A. Ostafe, I. E. Shparlinski, and J. F. Voloch (2023), have strengthened this quantitatively. A higher-dimensional generalization, again for almost all moduli, has also recently been studied by P. Kurlberg, A. Ostafe, Z. Rudnick, and I. E. Shparlinski (2024). In this talk, we discuss the problem over sequences of prime powers for fixed prime bases. This is based on a joint work with I. E. Shparlinski.

## Mridul Biswas

**Title:** Severi–Brauer Bundle Violating the Hasse Principle

**Abstract:** We study Severi–Brauer bundles and their Brauer groups. We answer a question posed by Bjorn Poonen in a more general setting by constructing Severi–Brauer bundles that fail the Hasse principle for rational points, without requiring explicit calculations. Based on a joint work with Divyasree C Ramachandran, B. Samanta.

**James Borger****Title:** Scheme theory over semirings

**Abstract:** Usual scheme theory can be viewed as the syntactic theory of polynomial equations with coefficients in a ring, most importantly the ring of integers. But none of its most fundamental ingredients, such as faithfully flat descent, require subtraction. So we can set up a scheme theory over semirings (“rings but possibly without additive inverses”, such as the non-negative integers or reals), thus bringing positivity in to the foundations of scheme theory. It is reasonable to view non-negativity as integrality at the infinite place, the Boolean semiring as the residue field there, and the non-negative reals as the completion.

In this talk, I’ll discuss some recent developments in module theory over semirings. While the classical definitions of “vector bundle” are not all equivalent over semirings, the classical definitions of “line bundle” are all equivalent, which allows us to define Picard groups and Picard stacks. The narrow class group of a number field can be recovered as the reflexive class group of the semiring of its totally nonnegative integers, i.e. the arithmetic compactification of the spectrum of the ring of integers. This gives a scheme-theoretic definition of the narrow class group, as was done for the ordinary class group a long time ago.

I’ll also give a number of basic open questions. I hope to say a word about applications.

This is based mostly on arXiv:2405.18645, which is joint work with Jaiung Jun, and also on forthcoming paper with Johan de Jong and Ivan Zelich.

**Florian Breuer****Title:** SeCritMass: Threshold Secret Petitions

**Abstract:** I will introduce the notion of an  $n$ -threshold secret petition, in which users add encrypted signatures to a petition, and the signatures are decrypted if and only if at least  $n$  signatures have been gathered. This solves the coordination problem in which users wish to sign a petition or commit to a cause, but do not want to be identified as having signed it before enough others have signed it too. I will present an implementation of such a petition based on the ElGamal cryptosystem

**Blair Butler****Title:** The Field of Definition of Shioda’s Rank 68 Elliptic Surface is a Number Field of Degree 829,440

**Abstract:** For an elliptic surface, the field of definition is the smallest number field where all of its sections are defined. We present the result, as well give a brief overview on the computation, that verify that the field of definition of Shioda’s Rank 68 elliptic surface given by  $y^2 = x^3 + t^{360} + 1$  is a number field which has degree 829,440.

**Sneha Chaubey****Title:** On the distribution of Kloosterman angles

**Abstract:** In this talk, we focus on Kloosterman sums, defined as  $K(q, a) = \sum_{x \in \mathbb{F}_q^*} e\left(\frac{x+ax^{-1}}{q}\right)$ , where  $e(t) = e^{2\pi it}$ . By choosing  $\theta_{q,a} \in [0, 1]$  such that

$$\cos \pi \theta_{q,a} = \frac{K(q, a)}{2\sqrt{q}},$$

we obtain the Kloosterman angles  $\theta_{q,a}$ . It is well known that as  $q \rightarrow \infty$ , the families

$$\mathcal{F}_q = \{\theta_{q,a}, 1 \leq a \leq q-1\}$$

are equidistributed in the interval  $[0, 1]$  with respect to the Sato-Tate measure  $\mu(t)dt$ , where  $\mu(t) = 2\sin^2(\pi t)$ . In the 1990s, Katz conjectured about the spacing statistics for the “straight-

ened” Kloosterman angles. Motivated by Katz’s conjecture, we study the pair correlation statistics for these angles. This is a joint work with R. Pal, S. Shabnam, and K. Sinha.

### Shashi Chourasiya

**Title:** An explicit form of Ingham’s zero density estimate for the zeta

**Abstract:** Ingham (1940) proved that  $N(\sigma, T) \ll T^{3(1-\sigma)/(2-\sigma)} \log^5 T$ , where  $N(\sigma, T)$  counts the number of the non-trivial zeros  $\rho$  of the Riemann zeta-function with  $\Re\{\rho\} \geq \sigma \geq 1/2$  and  $0 < \Im\{\rho\} \leq T$ . We discuss an explicit version of this result with the exponent 3 of the logarithmic factor. In addition, we also provide an explicit estimate with asymptotically correct main term for the fourth power moment of the Riemann zeta-function on the critical line. This talk is based on the joint work with A. Simonič.

### Raiza Corpuz

**Title:** Congruences of  $p$ -adic  $L$ -functions of modular forms at non-ordinary primes

**Abstract:** In this joint work with Antonio Lei, we present an analogue of Greenberg–Vatsal’s and Emerton–Pollack–Weston’s results on congruences of  $p$ -adic  $L$ -functions for  $p$ -non-ordinary cuspidal eigenforms  $f$  and  $g$  of equal weight that are  $p$ -congruent. In particular, we prove that the Iwasawa invariants of the analytic and algebraic signed  $p$ -adic  $L$ -functions of  $f$  and  $g$  are related by explicit formulae under appropriate hypotheses. We also show under the same assumptions that provided the algebraic and analytic  $\mu$ -invariants vanish, the signed Iwasawa main conjecture is true for  $f$  if and only if it is true for  $g$ .

### Chandler Corrigan

**Title:** Moments of restricted divisor functions (II)

**Abstract:** We derive asymptotic formulae for the higher-power moments of restricted divisor functions. In order to establish our main results, we study more general systems of multiplicative Diophantine equations and derive asymptotic and O-results in these directions. Joint work with Muhammad Afifurrahman.

### Yiannis Fam

**Title:** An exceptional L-packet in small residue characteristic.

**Abstract:** In 1876, Klein famously determined the finite subgroups of  $\mathrm{SO}_3(\mathbb{C})$  - they are cyclic, dihedral or the symmetry group of a platonic solid. When  $k$  is a non-archimedean local field, one is interested in the representations of the Weil-Deligne group of  $k$ . For inertially discrete representations, these correspond to finite Galois extensions of  $k$  with an embedding of the Galois group into a prescribed complex reductive group. In small residue characteristic, these Galois extensions of  $k$  can be more exotic, as we shall see for the group  $\mathrm{SO}_3(\mathbb{C})$ . In this talk I will describe similarly exotic behaviour in small residue characteristic for the exceptional group  $G_2$ .

### Steven Galbraith

**Title:** Isogeny graphs of elliptic curves and cryptographic applications

**Abstract:** The talk will survey some applications of isogenies in post-quantum public key cryptography. I will discuss isogeny volcanos in both the ordinary and supersingular case, and mention some open problems. I will present recent joint work with Valerie Gilchrist and Damien Robert on climbing isogeny volcanos. In particular, we give a generalised framework for some recent developments in self-pairings due to Castryck-Houben-Merz-Mula-van Buuren-Vercauteren and Macula-Stange.

**Peng Gao****Title:** Twisted fourth moment of Dirichlet L-functions to a fixed modulus**Abstract:** The fourth moment of Dirichlet L-functions at the central point to a fixed modulus has been extensively studied in the literature. Most of the work either concerns with the case when the moment is untwisted or when the fixed modulus is a prime. Given that the twisted moment has wide applications in bounding lower moments of the same family and in estimating the corresponding character sums, it is interesting to evaluate the twisted moment to a general modulus. In this talk, I will explain a result on an asymptotic formula with a power saving error term concerning the twisted four moment on the critical line of the family of Dirichlet L-functions to a fixed prime power modulus. This is a joint work with L. Zhao.**Michael Harm****Title:** Refinements for primes in arithmetic progressions.**Abstract:** Given a zero-free region and an averaged zero-density estimate for all Dirichlet L-functions modulo  $q$ , we refine the error terms of the prime number theorem in all and almost all short arithmetic progressions. For example, if we assume the Generalized Density Hypothesis, then for any arithmetic progression modulo  $q \leq \log^\ell x$  with  $\ell > 0$  and any  $\alpha > \frac{2}{3}$ , the prime number theorem holds in all intervals  $(x, x + \sqrt{x} \exp(\log^\alpha x)]$  and almost all intervals  $(x, x + \exp(\log^\alpha x)]$  as  $x \rightarrow \infty$ . This refines the classic intervals  $(x, x + x^{1/2+\varepsilon}]$  and  $(x, x + x^\varepsilon]$  for any  $\varepsilon > 0$ .**David Harvey****Title:** Integer multiplication is at least as hard as matrix transposition**Abstract:** It was recently proved that two  $n$ -bit integers may be multiplied in  $O(n \log n)$  steps on a multitape Turing machine. This bound is believed to be optimal, but no lower bounds have been established beyond the trivial  $\Omega(n)$  bound. In a paper to be presented at FOCS 2025 in Sydney later this year, Joris van der Hoeven and I give a reduction from the *transposition problem* for binary matrices to the integer multiplication problem. There is a simple folklore algorithm that transposes an  $n \times n$  binary matrix in time  $O(n^2 \log n)$ . Again, this is believed to be optimal, but no proof is known. Our new reduction implies that if this transposition algorithm is optimal, then integer multiplication satisfies the expected  $\Omega(n \log n)$  lower bound. In this talk I will give an overview of how the reduction works.**Harald Helfgott****Title:** Optimal bounds on sums of arithmetic functions**Abstract:** (joint with Andres Chirre) Let  $F(s) = \sum_n a_n n^{-s}$  be a Dirichlet series with meromorphic continuation. Say we are given information on the poles of  $F(s)$  with  $|\Im s| \leq T$  for some large constant  $T$ . What is the best way to use such finite spectral data to give explicit estimates on sums  $\sum_{n \leq x} a_n$ ?The problem of giving explicit bounds on the Mertens function  $M(x) = \sum_{n \leq x} \mu(n)$  illustrates how open this basic question was. Bounding  $M(x)$  might seem equivalent to estimating  $\psi(x) = \sum_{n \leq x} \Lambda(n)$  or the number of primes  $\leq x$ . However, we have long had fairly good explicit bounds on prime counts, while bounding  $M(x)$  remained a notoriously stubborn problem.We prove a sharp, general result on sums  $\sum_{n \leq x} a_n n^{-\sigma}$ , giving an optimal way to use information on the poles of  $F(s)$  with  $|\Im s| \leq T$ , with no need for zero-free regions. We obtain bounds on  $M(x)$  stronger than previous ones by many orders of magnitude, and also give new estimates for  $\psi(x)$ .

Our solution mixes a Fourier-analytic approach in the style of Wiener–Ikehara with contour-shifting, using optimal approximants of Beurling–Selberg type found in (Graham–Vaaler, 1981) and (Carneiro–Littmann, 2013). While we proceed independently of existing explicit work on

$M(x)$  and  $\psi(x)$ , our method has an important step in common with work on another problem (Ramana–Ramaré, 2020). For our applications, we use residue computations by D. Platt.

### Mumtaz Hussain

**Title:** The Generalised Baker–Schmidt Problem

**Abstract:** The Generalised Baker–Schmidt Problem (1970) concerns the  $f$ -dimensional Hausdorff measure of the set of well-approximable points on a nondegenerate manifold. There are two variants of this problem: simultaneous and dual approximation. The divergence cases have long been established for dual approximation on arbitrary nondegenerate manifolds. The corresponding convergence case remains a major and challenging open question. In this talk, I will walk through some recent progress for manifolds of co-dimension 1 and 2.

### Siddharth Iyer

**Title:** Gaps between quadratic forms

**Abstract:** Let  $\Delta$  denote the integers represented by the quadratic form  $x^2 + xy + y^2$  and  $\square_2$  denote the numbers represented as a sum of two squares. For a non-zero integer  $a$ , let  $S(\Delta, \square_2, a)$  be the set of integers  $n$  such that  $n \in \Delta$ , and  $n + a \in \square_2$ . We conduct a census of  $S(\Delta, \square_2, a)$  in short intervals by showing that there exists a constant  $H_a > 0$  with

$$\#S(\Delta, \square_2, a) \cap [x, x + H_a \cdot x^{5/6} \cdot \log^{19} x] \geq x^{5/6-\varepsilon}$$

for large  $x$ . To derive this result and its generalization, we utilize a theorem of Tolev (2012) on sums of two squares in arithmetic progressions and analyse the behavior of a multiplicative function found in Blomer, Brüdern & Dietmann (2009). Our work extends a classical result of Estermann (1932) and builds upon work of Müller (1989).

### Daniel Johnston

**Title:** Do Geese see God? An exploration of recent results on Palindromic Numbers.

**Abstract:** Palindromic numbers, such as 787 or 122333221, have been central objects in recreational mathematics for a long time. It is still unknown whether, in every fixed base  $b > 1$ , there are infinitely many palindromic numbers which are also prime. In this talk, we explore some very recent results in this area, whereby new progress seems to be made every month.

### Shin-ya Koyama

**Title:** Chebyshev’s bias and the Deep Riemann Hypothesis

**Abstract:** Let  $\pi(x, q, a)$  be the number of primes  $p < x$  such that  $p \equiv a \pmod{q}$ . Chebyshev’s bias is a phenomenon that the inequality  $\pi(x, q, b) > \pi(x, q, a)$  holds more often than not for  $a \in (\mathbb{Z}/q\mathbb{Z})^{\times 2}$  and  $b \in (\mathbb{Z}/q\mathbb{Z})^\times \setminus (\mathbb{Z}/q\mathbb{Z})^{\times 2}$ . It was formulated by Rubinstein and Sarnak (1994) in terms of the logarithmic measure of  $x > 0$  such that the inequality holds.

In 2023 Aoki and I proposed a new definition of Chebyshev’s bias by using the weighted counting function

$$\pi_{\frac{1}{2}}(x, q, a) = \sum_{\substack{p \equiv a \pmod{q} \\ p \leq x}} p^{-\frac{1}{2}}.$$

Indeed we defined the Chebyshev bias by the asymptotic

$$\pi_{\frac{1}{2}}(x, q, b) - \pi_{\frac{1}{2}}(x, q, a) = c_q \log \log x + O(1) \quad (x \rightarrow \infty)$$

with an explicit constant  $c_q$ . We proved that the bias exists under the assumption of the Deep Riemann Hypothesis for Dirichlet  $L$ -functions, which asserts the convergence of the Euler products on the critical line.

In this talk we introduce generalizations and refinements of this new formulation. We also discuss refinements by the contribution from nontrivial zeros of Dirichlet  $L$ -functions.

### Victor Lu

**Title:** 16-Descent on Elliptic Curves

**Abstract:** The method of descent for Diophantine equations involves deriving auxiliary equations which may contain information on the solutions of the original equation. In modern number theory descent is used to find potential rational points of Abelian varieties. For elliptic curves over number fields  $K$ , algorithms for explicit  $p^n$ -descent for  $p = 2$  and  $n = 1, 2, 3$  has been developed. We aim to extend it to  $n = 4$  case, which would allow us to find better bounds for the group of rational points  $E(K)$  when the Tate-Shafarevich group of  $E/K$  has nontrivial 2-primary part. In this talk we describe the general theory behind descent, and some of the work we've done so far for 16-descent on elliptic curves over number fields.

### Riddhi Manna

**Title:** Sun's conjecture on the summatory function of  $(-2)^{\Omega(n)}$

**Abstract:** A conjecture of Sun states that the function

$$W(x) = \sum_{n \leq x} (-2)^{\Omega(n)}$$

can be bounded as  $|W(x)| \leq x$  for all  $x \geq 3078$ . Here  $\Omega(n)$  denotes the number of prime factors of  $n$  counted with multiplicity. The Sun's conjecture was verified by Mossinghoff and Trudgian for  $x < 2.5 \times 10^{14}$ . Merten's conjecture is of a similar. The problem can be approached using analytic techniques. We consider the function

$$\sum_{n \leq x} (-2)^{\Omega(n)} \log \left( \frac{x}{n} \right)$$

to introduce a degree of smoothness to the sum. Using techniques of contour integration and a variant of Perron's formula, this sum is then estimated.

### Alina Ostafe

**Title:** On the Skolem Problem for parametric families of linear recurrences

**Abstract:** In this talk I will discuss recent results about the Skolem problem for specialisations of linear recurrences defined over a function field. More precisely, given rational functions  $a_1, \dots, a_k, f_1, \dots, f_k$  defined over a number field, for all but a set of elements  $\alpha$  of bounded height in the algebraic closure of  $\mathbf{Q}$ , the Skolem problem is solvable for the linear recurrence

$$F_n(\alpha) = a_1(\alpha)f_1(\alpha)^n + \dots + a_k(\alpha)f_k(\alpha)^n, \quad n \geq 0,$$

and the existence of a zero can be effectively decided. Moreover, for linear recurrences of order at most 3 we have finiteness results for specialisations at roots of unity that give a zero in such a sequence. If time allows I will also briefly discuss connections to certain gcd problems for linear recurrences.

These are joint works with Philipp Habegger, David Masser and Igor Shparlinski.

### Neea Palojärvi

**Title:** On Turing's method for Artin  $L$ -functions and the Selberg class

**Abstract:** A famous conjecture, the Riemann hypothesis, states that all of the non-trivial zeros of the zeta function lie on the line  $\Re(s) = 1/2$ . The Generalized Riemann hypothesis states the same for  $L$ -functions. These conjectures seem to be out of reach of current mathematical

knowledge. Hence, a natural follow-up question is to ask for partial results, for instance, does the hypothesis hold up to some height. These types of results are very important in explicit number theory since usually the zeros with small absolute values affect the explicit results more than those with large absolute values.

A common approach to detect zeros of the Riemann zeta function up to some height is based on Turing's method, developed by A. Turing in 1953. The essential idea is to find a list of zeros and then check that all of them have indeed been found. In this talk, I will discuss Turing's method for a large set of  $L$ -functions consisting of Artin  $L$ -functions and a certain subset of the Selberg class functions. The talk is based on work with Tianyu Zhao, and is a continuation of A. R. Booker's (2006) research.

### **Fabien Pazuki**

**Title:** Isogeny volcanoes: an ordinary inverse problem

**Abstract:** We prove that any abstract  $\ell$ -volcano graph can be realized as a connected component of the  $\ell$ -isogeny graph of an ordinary elliptic curve defined over  $\mathbb{F}_p$ , where  $\ell$  and  $p$  are two different primes. If time permits, we will touch upon some new applications and new challenges. This is joint work with Henry Bamby and Francesco Campagna.

### **Derek Perrin**

**Title:** Ordinary isogeny graphs with level structure

**Abstract:** We study  $\ell$ -isogeny graphs of ordinary elliptic curves defined over  $\mathbb{F}_q$  with an added level structure. Given an integer  $N$  coprime to  $p$  and  $\ell$ , we look at the graphs obtained by adding  $\Gamma_0(N)$ ,  $\Gamma_1(N)$ , and  $\Gamma(N)$ -level structures to volcanoes. Given an order  $\mathcal{O}$  in an imaginary quadratic field  $K$ , we look at the action of generalised ideal class groups of  $\mathcal{O}$  on the set of elliptic curves whose endomorphism rings are  $\mathcal{O}$  along with a given level structure. We show how the structure of the craters of these graphs is determined by the choice of parameters.

### **Jordy Pertile**

**Title:** Sharper Explicit Bounds for Discriminants via Stark's Lemma

**Abstract:** We explore explicit lower bounds for the discriminant of number fields, focusing on refinements of Stark's lemma. By analysing the interplay between the Dedekind zeta function and analytic techniques developed by Odlyzko, we derive a computable inequality for the discriminant. Our main result improves the error term in classical results and yields a fully explicit inequality. In this talk we will discuss the strategy of the proof, the role of the digamma and zeta functions, potential applications, and future work.

### **Chao Qin**

**Title:** From Gauss to Bianchi: An Algorithm for Computing Class Representatives of Binary Hermitian Forms

**Abstract:** The reduction theory of integral binary quadratic forms, as pioneered by Gauss, provides a canonical representative for each equivalence class of forms under the action of  $\mathrm{SL}_2(\mathbb{Z})$ . This classical theory has a well-known geometric interpretation involving the action of the modular group on the upper half-plane. This talk will first revisit these foundational concepts before turning to the generalization initiated by Hermite and Bianchi: the study of binary Hermitian forms over the ring of integers  $\mathcal{O}$  of an imaginary quadratic field. In this setting, the problem becomes the classification of forms under the action of the Bianchi group  $\mathrm{PSL}_2(\mathcal{O})$  on hyperbolic 3-space. The central task is to find a complete set of representatives for the equivalence classes of positive definite binary Hermitian forms of a given discriminant  $\Delta$ . We will present a new algorithm that explicitly computes these class representatives. This is based on ongoing joint work with Di Zhang.

**Maurice Rojas****Title:** New Quantitative Bounds for  $p$ -adic Fewnomials

**Abstract:** While 17th century work of Descartes yields a tight upper bound of  $t - 1$  for the number of positive roots of a univariate  $t$ -nomial, the analogous bounds over the  $p$ -adic rationals, due to Lenstra, Avendano, and Krick (after 1997) are not yet known to be tight. Even less is known for systems of  $n$  polynomial equations in  $n$  unknowns over  $\mathbb{Q}_p$ , despite work of Denef and van den Dries around 1988. So we present the first explicit upper bounds for the number of isolated roots (over  $\mathbb{Q}_p$ ) of  $n$  by  $n$  systems having a total of  $n + 2$  distinct exponent vectors, and discuss some related tropical geometric aspects. This is joint work with Joshua Goldstein, Henry Stone, and Arnaldo Vera.

**Simon Rutard****Title:** Analytic properties of Witten zeta functions

**Abstract:** Originally introduced in mathematical physics, Witten zeta functions have recently gained interest due to their connection with the asymptotic behavior of  $n$ -dimensional representations of Lie groups. Their special values reflect some underlying properties of the group, and a recent result by Au (2024) proves that these functions vanish at negative even integers when the group is simply connected. In this talk, we will explain a generalization of this result to non-simply connected Lie groups. We will also discuss the broader class of Witten L-functions and their special values.

**Igor Shparlinski****Title:** Counting Questions for Linear Recurrence Sequences and  $S$ -unit equations

**Abstract:** We consider several statistical questions about zeros in terms of linear recurrence sequences (LRS) or terms factored from primes from a given set of primes. For example, we will give tight both-sided bounds on the frequency of LRS that have zeros amongst their terms. These questions are motivated by the still open Skolem problem. We will also consider multiplicative dependence in  $s$ -tuples of terms of LRS. Finally, we will show a rich variety of techniques behind these results and formulate several open questions.

**Alexei Skorobogatov****Title:** The Brauer group of an abelian variety

**Abstract:** The importance of the Brauer group for arithmetic geometry was highlighted by Manin in his celebrated 1970 ICM address. In this talk I will discuss the structure of the Brauer group of an abelian variety  $A$  over an algebraically closed field of characteristic  $p$  focusing on the  $p$ -primary torsion, the key part of which is a certain quasi-algebraic unipotent group  $U_A$ . I will present results on the dimension and the  $p$ -exponent of  $U_A$  based on the classical Manin-Dieudonné theory, leading to the determination of  $U_A$  up to isogeny for abelian varieties  $A$  of small dimension. This is joint work with Livia Grammatica and Yuan Yang.

**László Szalay****Title:** On  $p$ -periodic linear recurrences

**Abstract:** We study the so-called  $p$ -periodic homogeneous linear recurrences in general, without any restriction on the order of recursive formulae and on the number of branches. If we denote such a sequence by  $\{G_n\}_{n \geq 0}$ , then the expression  $p$ -periodic means that the recursive formula for the term  $G_n$  depends on the smallest non-negative remainder  $i$  modulo  $p$ , where  $p \geq 2$  is a fixed integer. That is

$$G_n = a_{i,1}G_{n-1} + \cdots + a_{i,k}G_{n-k} \quad \text{if } n \equiv i \pmod{p}, \quad (0 \leq i \leq p-1). \quad (1)$$



Here the coefficients  $a_{i,j}$  are fixed complex numbers, and  $k$  is a positive integer.

We extend many results have been felt by presenting a general approach to provide an unconditional recursive formula for  $G_n$ . The method is demonstrated by some old and new problems of the subject material.

The results are joint work with M. Rachidi and F. Yilmaz.

### Valerio Talamanca

**Title:** On Symmetries of Height Functions Associated to a Class of Representations of  $\mathbb{G}_m^d$ .

**Abstract:** Let  $k$  be a number field and let  $\mathbb{G}_m^d$  denote the  $k$ -split algebraic torus of dimension  $d$ . Let  $E$  be a  $\mathbb{G}_m^d$ -module and let  $\rho : \mathbb{G}_m^d \rightarrow \mathrm{GL}(E)$  be the associated homomorphism. The height  $h_E$  associated to  $E$  is defined by setting  $h_E := h_s \circ \rho$ , where  $h_s$  denotes the spectral height on  $\mathrm{GL}(E)$ . In this talk we discuss the group of symmetries for  $h_E$ , when  $E$  belongs to a class of representations.

### Simon Thomas

**Title:** Almost primes between all cubes

**Abstract:** It is known that there exists a prime between  $n^3$  and  $(n+1)^3$  for sufficiently large  $n$ , but this result has not yet been extended to all  $n \geq 1$ . We discuss progress towards a variant of this problem where instead of primes, we look for numbers with at most 2 prime factors. Dudek and Johnston recently proved an analogous result for squares using an explicit linear sieve along with some computational efforts, and we hope that a similar approach will prove fruitful here. Johnston is back for more in this ongoing joint work, alongside Sorenson and Webster.

### Lola Thompson

**Title:** Sums of proper divisors meet integers with missing digits

**Abstract:** Let  $s(n)$  denote the sum of proper divisors of an integer  $n$ . The function  $s(n)$  has been studied for thousands of years, due to its connection with the perfect numbers. In 1992, Erdős, Granville, Pomerance, and Spiro (EGPS) conjectured that if  $\mathcal{A}$  is a set of integers with asymptotic density zero then  $s^{-1}(\mathcal{A})$  also has asymptotic density zero. This has been confirmed for certain specific sets  $\mathcal{A}$ , but remains open in general. In this talk, we will give a survey of recent progress towards the EGPS conjecture. In particular, we will examine whether the conjecture holds for different sets of integers with missing digits. This talk is based on joint work with Paul Pollack and Carl Pomerance, and also on joint work with Kübra Benli, Giulia Cesana, Cécile Dartyge, and Charlotte Dombrowsky.

### Cai Tianxin

**Title:** Some number theory problems related to binomial coefficients

**Abstract:** In this talk, I will first introduce some kinds of congruences of Lehmer-Morley-Jacobstahl modulo integer powers and their links to the three types of complete elliptic integrals. Then I will use binomial coefficients in the representation of prime numbers by binary forms. Finally, I define figurate primes by binomial coefficients, and use them in Hilbert's 8th problem.

### Tim Trudgian

**Title:** Make ANTEDB Great Again!

**Abstract:** I would have spelled out the acronym, the Analytic Number Theory Exponent Database, but that wouldn't fit on our fetching red caps! I will talk about some recent work with Terry Tao and Andrew Yang that combines number theory with formal proofs in Lean and such like. The goal is to build a grassroots movement of contributors to the noble cause,

github.com/teorth/expdb, so that, together, we can Make Analytic Number Theory Great Again!

**Sebastian Tudzi**

**Title:** Certain Arithmetic Functions Involving the G.C.D.

**Abstract:** In this talk, we will explore the asymptotic of a class of arithmetic functions that describe the value distribution of the greatest-common-divisor function. These functions are typically generated by a Dirichlet series whose analytic properties are governed by the factor  $\zeta^2(s)\zeta(2s-1)$ .

**Ilaria Viglino**

**Title:** Moment estimates of module lattice points for effective lattice constructions

**Abstract:** We examine the moments of the number of lattice points in a fixed ball of volume  $V$  for lattices in Euclidean space which are modules over the ring of integers of a number field  $K$ . In particular, we show that moments obtained for “lifts of codes” to  $\mathcal{O}_K$ -modules converge to the Rogers integral formula for the space of free  $\mathcal{O}_K$ -module lattices. This extends work of Rogers for  $\mathbb{Z}$ -lattices. Joint work with Maryna Viazovska, Nihar Gargava and Vlad Serban.

**Liang Wang**

**Title:** Metric Poissonian pair correlation for real sequences

**Abstract:** Poissonian pair correlation is a local statistic that captures strong pseudo-randomness in deterministic sequences. In recent joint work with Bryce Kerr, we establish new sufficient conditions for the metric Poissonian property. As applications, we show that both convex and polynomial sequences are metric Poissonian.