The Tate-Voloch conjecture

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Conjecture

Let K be a field with a non-archimedian absolute value, A/K be a semiabelian variety and X a closed subvariety. There is a lower bound $c > 0$ for the distance (obtained from the absolute value on K) of torsion points on A, not in X, to X.

We stated this conjecture and proved it in the case A is a torus and $K = \mathbb{C}_p$. (IMRN 1996)

Results

The conjecture was proved by T. Scanlon (1999) when $K = \bar{\mathbb{Q}}_p$.

An analogue for Shimura varieties (with CM points replacing torsion points) was considered by P. Habegger (2014) and Q. Liu (2023).

An analogue, when A is a torus but X is an analytic subvariety of A was considered by A. Neira (2002) and V. Serban (2018).

Analogues where the torsion points are replaced by preperiodic points of an endomorphism of an arbitrary ambient variety A were considered by W. Peng (2023), J. Xie (2018), Ih (2011), Ih and Benedetto (2024).

THANK YOU