The ‘Sober’ way to drink tequila

A curious result:

There are three statistically independent causes that can have two effects, so that:

1. For each cause, the two effects become statistically independent once we learn whether or not that cause occurs.
2. The two effects also become statistically independent once we learn exactly which of the three causes occur.
3. But, for any two causes, the two effects remain statistically dependent when we learn which of the two causes occur.

The set-up: A bar game involves three players, each of whom reveals either a clenched fist or an open hand; they do so simultaneously, by the usual ‘three shakes method’.

A person ‘wins’ this game if he or she reveals a hand that differs from what the other two players present, in which case the latter two ‘lose’. Otherwise (i.e., if all three reveal a clenched fist, or all reveal an open hand) the game is a ‘draw’.

Suppose each player decides his or her strategy independently, and chooses between the two strategies with equal probability. If there is a winner, this player must drink a glass of tequila, while each of the two losers is required to toss a fair coin, and drink a glass of tequila precisely if the toss lands heads. If the game ends in a draw, then each player independently rolls a fair six-sided die and if the number that comes up is a 3 or a 6 that person drinks a glass of tequila.

Let $E_1$ be the event that player 1 drinks a glass of tequila, and $E_2$ the event that player 2 drinks a glass of tequila (we aren’t interested here in whether player 3 partakes). Then take $E_1$ and $E_2$ to be two effects, and let the three causes to be the hand actions of the three players. Then properties 1, 2, 3 described above all apply.