SPECIAL ISSUE IN HONOUR OF JAMES OXLEY

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This special issue is dedicated to Professor James Oxley to celebrate his sixtieth birthday in February 2013. Most of the papers are on topics in matroid theory, a subject to which James has contributed so much in his research career.

1. Brief Biography

James Oxley was born in Sale, Victoria, Australia in February 1953. After Mentone Grammar School in Melbourne, he graduated with First Class Honours in Mathematics from the University of Tasmania. From there, he moved to the Australian National University and, in 1975, completed an MSc with a thesis entitled 'Regular Combinatorial Polyhedra' under the supervision of Michael Newman. This led naturally to his doctoral work at Oxford, where, after three years under the supervision of Dominic Welsh, he obtained his doctorate with a thesis entitled 'Some Problems in Combinatorial Geometry'. This could be described as matroid theory in disguise.

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While still a graduate student at Oxford, he took a position as Visiting Instructor at the University of North Carolina, Chapel Hill, working with the late Tom Brylawski.

After completing his doctorate in 1978, he took up a three-year position as Lecturer/Research Fellow at the Australian National University (ANU). Whilst there, he was awarded a Fulbright Postdoctoral Fellowship at the University of North Carolina, Chapel Hill. There he worked with Tom Brylawski and Doug Kelly, two leading researchers in matroid theory.

In 1982, he took up a position as Assistant Professor of Mathematics at Louisiana State University, where he has remained to this day. He was promoted to full Professor in 1990. He has been elected to a Visiting Fellowship at Merton College, Oxford (2005) and to a Visiting Erskine Fellowship at the University of Canterbury (2010). James' outstanding academic contributions, both as a teacher and as a researcher, have been recognised by the award of a prestigious Boyd Professorship by Louisiana State University in 2012. This award is the highest rank awarded by LSU and James was only the seventieth person to receive a Boyd Professorship since it was instituted in 1953.

On a personal side, apart from mathematics, one of James' main interests is sport. In particular, he is an unwavering supporter of the Australian cricket team. One notable example of his devotion to this sport occurred while he was a doctoral student at Oxford. He took two days off to travel to Leeds, where Australia were playing England. He returned slightly chastened and, for James, relatively quiet, having spent almost all of those two days watching Geoffrey Boycott bat for England. For those readers unfamiliar with cricket, Boycott was an outstanding player. However, watching him bat has sometimes been likened to watching paint dry.

Turning to James' research and scholarship, he has written more than 130 research papers, almost all on some aspect of matroid theory. After a brief introduction to matroids, we will highlight just a few pieces of work.

2. What is a Matroid?

The concept of a matroid was introduced by Hassler Whitney in 1935 as a generalisation or abstraction of both a graph and a collection of vectors. He gave several equivalent axiomatizations, and perhaps the most natural is the following: A collection of subsets of a finite set E forms the set of independent sets of a matroid if

(a) the empty set is independent,

(b) any subset of an independent set is independent, and
(c) given two independent sets of differing size, there is some element in the larger set and not in the smaller such that when added to the smaller preserves its independence.

The edge sets of a graph and the linearly independent subsets of a set of vectors clearly satisfy these axioms. Matroids isomorphic (defined in the obvious way) to a matroid obtained in this way from a graph or set of vectors are called, respectively, graphic and representable. Although these two classes form but a tiny proportion of all matroids they have served to motivate many of the crucial concepts. Deletion and contraction of an edge from a graph can be generated very easily to matroids, where they commute as in graph theory and lead to the fundamental substructure of a matroid, namely a minor. It is relatively straightforward to decide if a given matroid is graphic, but deciding representability over a given field is in general still a very difficult problem and one which is probably the most important area of research in matroid theory.

3. James' Contributions to Matroid Theory

One of James' earliest, and most important, pieces of work [4] was published in The Proceedings of the London Mathematical Society when he was still a graduate student. The substantial paper 'Infinite Matroids' was one of the earliest attempts to extend the classical axiom systems proposed by Hassler Whitney to the situation where the underlying ground set is infinite. This paper has received less attention than it deserves, probably because most combinatorialists like working with finite structures. It could also be because the subject matter is seriously hard.

A second example of his scholarship and expository ability is 'The Tutte Polynomial and its Applications' which he wrote with Tom Brylawski [1], and which appeared as a chapter in Matroid Applications published by Cambridge University Press in 1992. It is over one hundred pages long and even today is the standard reference on the many facets of the Tutte polynomial.

Throughout his research career, James has been interested in questions of representability and connectivity. An excluded-minor condition for a matroid to be binary (representable over $GF(2)$) was given by Tutte [9] and is the only fairly easy representability result known. In the same paper, Tutte also gave an excluded-minor characterisation of the class of regular matroids, that is, the matroids representable over all fields. Seymour [8] gave a structural characterisation of this class. In 1987, James described the structure of binary matroids with no 4-wheel minor and ternary matroids with no K_4 (or 3-wheel) minor [6]. As with most structural characterisations of minor-closed classes, these are highly non-trivial.

Of his many papers on connectivity, we highlight just one, partly because it is very attractive and also because it is relatively easy to explain. This paper [3], published in 2001 jointly with Manoel Lemos, gives a sharp upper bound on the maximum size of a connected matroid. Obviously, as stated, there exist connected matroids of arbitrarily large size. However, what James and Manoel show is that given the maximum size of a circuit in M and its dual M^*, the ground set of M is bounded by essentially the product of these maximum sizes.

A recent very important piece of research in contained in his 2004 paper with Choe, Sokal, and Wagner [2]. In this groundbreaking work of nearly one hundred pages, the authors exhibit a most surprising relationship between matroids and multivariate polynomials. A multivariate polynomial f in n complex variables is said to have the half-plane property if there is an open half plane H whose boundary contains the origin and f is non-zero whenever the variables are in H. They prove that the support (set of non-zero coefficients) of a multi-affine, homogeneous polynomial with the half-plane property is the set of bases of a matroid. For a while it was believed that the converse was true, unfortunately this is not the case. Nevertheless, the class of matroids which is (known as HPP matroids) is a fascinating, but not well-understood class.

Finally, we come to James' magnum opus 'Matroid Theory'. The first edition published in 1992 is an amazing, comprehensive volume. Over five hundred pages in length, it is an outstanding example of James' expository skills. It can be used as a text with which to learn matroids at the graduate level and also as the authoritative work in which to check results or conjectures. A particularly appealing feature is an appendix containing representations of the interesting matroids on up to 13 elements, many of which are given in a geometrical form even when graphical, for example, the matroid of the graph K_5 is shown as the 3-dimensional Desargues configuration. Another attractive feature of the book is that most proofs are given in full rather than merely having the hard parts "left to the reader". Few, if any, of those working in matroid theory could have written such a volume, but, amazingly, in 2011 James produced a second edition [7] of nearly seven hundred pages. This is genuinely new as, although it contains all the good features of the first edition, it includes all the important results obtained in the intervening years. This surely is going to be the classic reference point for matroid theorists for many years to come.

We hope that the above and the contents of this issue go some way towards conveying the importance of James' contribution to matroid theory over the past 30 plus years. Few have had such an impact, and it is hoped that his influence and research will continue for many more years.

Acknowledgements

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