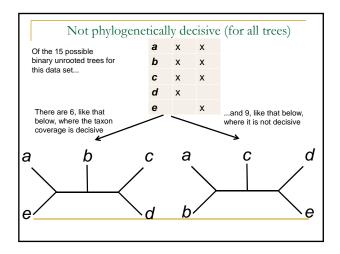
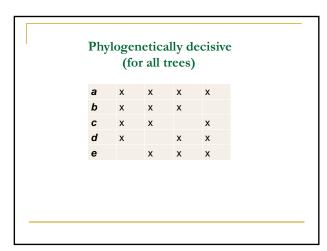
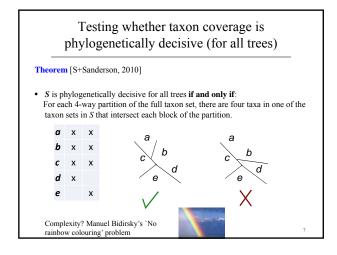


Definition: Let S be a collection of taxon sets. Then:
S is decisive for a phylogenetic tree T provided T is uniquely determined by how it resolves each of the taxon sets in S.
S is phylogenetically decisive for all trees if this holds for all choices of T.





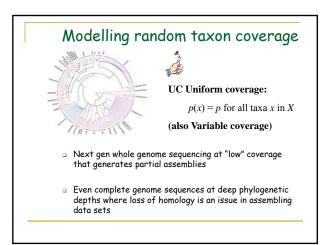


A lower bound on number of loci for decisiveness for all trees

Theorem:

If a collection S of taxon sets of size $n_1, n_2, ..., n_k$ is phylogenetically decisive for all trees with n leaves then: $\sum_{j=1}^k n_j (n_j - 1)(n_j - 2) \ge n(n-1)(n-2)$ So, if each taxon set in S has size at most m then: $k \ge \frac{n(n-1)(n-2)}{m(m-1)(m-2)} \ge (n/m)^3$ Examples:

max col density (m/n) "min" num loci (k)33% 27
80% 2

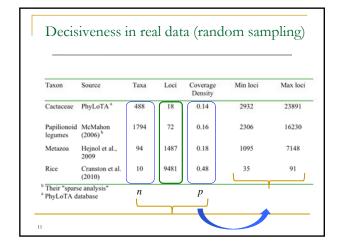


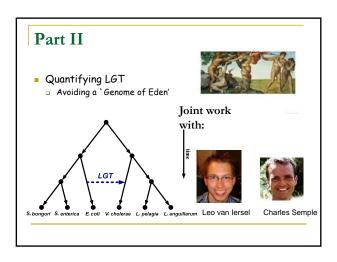
Theorem

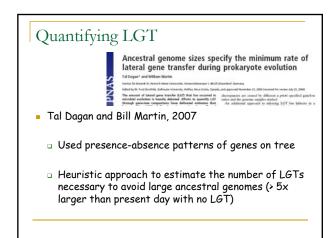
★ For any rooted binary tree T with n leaves, with coverage probability p, the probability that a set S of k (random) taxon sets is phylogenetically decisive for T is at least 1 - ε if

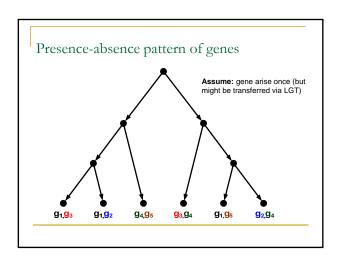
$$k \ge \frac{\log((n-2)/\varepsilon)}{-\log(1-p^3)}$$

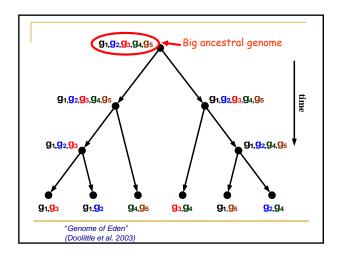
Moreover, if k is much less than this, then S is not (w.p. $1-\varepsilon$) phylogenetically decisive

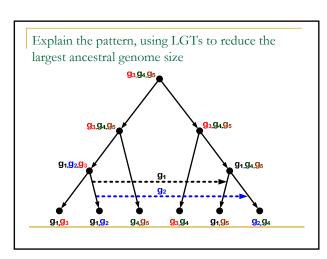












Without LGT, is there a tree that avoids a 'genome of eden'?

More precisely:

Given genomes is there a **tree** that can explain the pattern of presence/absence of genes **without any LGT** and insisting that no ancestral sequences are larger than k?



Theorem

This problem is already (NP) hard!

Even when |G(x)|=2 for all x.

What if the tree+LGT arcs (network) is given?

Given genome assignment to the leaves of a LGT network can we determine if ancestral genomes need to be larger than k?



Theorem

This problem is also (NP) hard!

Is there any good news?

1. Bounds:

For each species a collection of genes, max genome size k, and a tree T.

Find: Upper and lower bounds on the minimal number l(T,G,k) of LGT transfer events* needed.

Theorem

$$\begin{split} &\ell(T,G,k) \geq \sqrt{\frac{2}{3}|\{v \in V : n(v) > k\}|}.\\ &\ell(T,G,k) \leq \lceil \frac{|G|-k}{k} \rceil \cdot (|\mathcal{X}|+1). \end{split}$$

n(v):= number of genes g for which v is a mrca of two leaf taxa having g in their genomes *Each arc can transfer several genes!

2. An algorithm:

There is an efficient method for determining whether ancestral genomes of size >k can be avoided with at most m LGTs that form non-overlapping cycles

