STAT221

Tutorial 8: Practice questions

1. What is the size of the sample space for the experiment of rolling two regular hexagonal cylinders (with six identifiable faces each) three times in a row.

   Think first about the sample space of an experiment rolling one such cylinder. Then the sample space of rolling two of them. Then what happens if you do that three times.

2. Let $A = \{a, b, c, d, e\}$ be the set of offspring or children of the set of parents $B = \{\alpha, \beta, \gamma, \delta\}$. One of many possible offspring-parent relations could be summarised by the following set of ordered pairs:
$$\{(a, \alpha), (b, \alpha), (c, \beta), (e, \gamma), (d, \delta)\} \ .$$

   (a) Think of another set of ordered pairs that constitutes a function from A to B

   (b) Think of a set of ordered pairs that does **not** constitute a function from A to B. Explain why not.

3. Consider the following statements:

   (a) A statistic is a function of the data.

   (b) The data is a statistic.

   (c) An estimator is a statistic.

   (d) The empirical distribution function is a statistic.

   Choose the true answer:

   (A) Only (a) & (b) are true.

   (B) (a), (b), (c) & (d) are true.

   (C) (a) & (b) are true but (c) & (d) are false.

   (D) Only (c) is false.

4. Which one of the following statements about events $A_1, A_2, \ldots, A_n$ is true **in general**?

   (A) $P(A_1 \cup A_2 \cup \cdots \cup A_n) = P(A_1) + P(A_2) + \cdots + P(A_n)$

   (B) $P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2)$

   (C) $P(A_1 \cup A_2) = P(A_1) + P(A_2)$

   (D) $P(A_1 \cap A_2 \cdots \cap A_n) = P(A_1)P(A_2) \cdots P(A_n)$

5. Consider the following statements about two events A_1 and A_2 when $P(A_1) > 0$, $P(A_2) > 0$:

   (a) $P(A_1 \cap A_2) = P(A_1|A_2)P(A_2)$

   (b) $P(A_1 \cap A_2) = P(A_2|A_1)P(A_1)$

   (c) $P(A_1 \cap A_2) = P(A_1) + P(A_2)$

   (d) $P(A_1 \cap A_2) = P(A_1)P(A_2)$

   Which one of the following is correct, in general (without knowing more about A_1, A_2)

   (A) (a), (b), (c), (d) are all true in general (without knowing more about A_1, A_2)

   (B) (a) and (b) and (c) are all true in general; (d) is only true in particular circumstances.

(C) (a) and (b) are both true in general; (d) is only true in particular circumstances.

(D) (a) and (d) are both true in general; (b) is only true in particular circumstances

6. Consider the mutually exclusive events A and B with non-zero probabilities ($P(A) > 0$), $P(B) > 0$). Which of the following is **not** true?

   (A) $P(A|B) = 0$

   (B) $P(B|A) = 0$

   (C) $P(A \cap B) = P(A)P(B)$

   (D) $P(A \cup B) = P(A) + P(B)$

7. Which **one** of the following distributions does **not** correspond to a discrete RV?

   (A) A $Bernoulli(\theta)$ RV.

   (B) The sum of two IID $Bernoulli(\theta)$ RVs

   (C) An Exponential(λ) RV.

   (D) An equi-probable de Moivre($k = 4$) (de Moivre($\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}$)) RV

8. Let u be a sample from Uniform$(0, 1)$ RV and in our inversion sampling algorithm we have $x \leftarrow \pi + \pi u$. Then, x is a sample from:

   (A) $Uniform(-\pi, \pi)$ RV.

   (B) $Uniform(\pi, 2\pi)$ RV.

   (C) $Uniform(0, \pi)$ RV.

   (D) $Uniform(\pi/2, \pi)$ RV.

9. Which one of the following codes produces a sample $x \in 1, 2, 3, 4$ from an equi-probable de Moivre($k = 4$) (de Moivre($\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}$)) RV?

   (A)
   ```
   u = random()
   x = 4*ceil(u) + 1
   ```

   (B)
   ```
   u = random()
   x = ceil(u) + 4
   ```

   (C)
   ```
   u = random()
   x = 4*floor(u) + 1
   ```

   (D)
   ```
   u = random()
   x = floor(u*4) + 1
   ```

10. Which of the following RVs is **not** the sum of the outcome of rolling two fair dice?

   (A) de Moivre$(0, 1/36, 1/18, 1/12, 1/9, 5/36, 1/6, 5/36, 1/9, 1/12, 1/18, 1/36)$

   (B) de Moivre$(1/36, 1/18, 1/12, 1/9, 5/36, 1/6, 5/36, 1/9, 1/12, 1/18, 1/36) + 1$

   (C) $X_1 + X_2$, where $X_1, X_2 \overset{IID}{\sim}$ de Moivre$(1/6, 1/6, 1/6, 1/6, 1/6, 1/6)$

(D) de Moivre(1/12, 1/12, 1/12, 1/12, 1/12, 1/12, 1/12, 1/12, 1/12, 1/12, 1/12, 1/12)

11. Let X be a random variable (RV) with possible values in the set $\{0, 1, 2, 3\}$ and with corresponding probabilities $\{0.1, 0.2, 0.3, 0.4\}$. What is the expectation or expected value of X^2 ?

    (A) 2

    (B) 3.5

    (C) 5

    (D) 9

    SCENE for the next two questions:
    An assembly plant receives its components from 3 suppliers: 60% from B_1, 30% from B_2 and 10% from B_3. 95% of the components from B_1, 80% from B_2 and 65% from B_3 perform according to specifications.

12. What is the probability that a component will perform according to specifications?

    This is quite a tough question. First think about what you have been told in terms of conditional probabilities (probability that a component works given that it comes from a particular supplier). Then, think about how you can use the general formula $P(A \cap B) = P(A|B)P(B)$ to get the probability that a component works and comes from supplier B_1, and the same for the probability that a component works and comes from supplier B_2, and the same for the probability that a component works and comes from supplier B_3. Then think about how you can use the rule for disjoint events (if $E_1 \cap E_2 = \emptyset, P(E_1 \cap E_2) = 0, P(E_1 \cup E_2) = P(E_1) + P(E_2)$) to get the probability that the component works as the probability of the union of disjoint events.

    (A) 650/1000

    (B) 800/1000

    (C) 875/1000

    (D) None of the above

13. Given that a component performs according to specifications, what is the probability that it came from supplier B_2?

    Think about $(B|A) = \frac{P(A \cap B)}{P(A)}$

    (A) 240/1000

    (B) 240/875

    (C) 535/875

    (D) 1/3

14. In the Assessment2a we considered the random variable X that is a discrete mixture of two Uniform(θ_1, θ_2) RVs. The PDF of X can be expressed as follows:

$$f(x) = \frac{1}{2} 3 \, \mathbb{1}_{[0,1/3]}(x) + \frac{1}{2} 3 \, \mathbb{1}_{[2/3,1]}(x)$$

    The RV X above can be constructed from the following process. I flip a fair coin and if I get heads then I choose a real number uniformly at random from $[0, 1/3]$ and if I get tails then I choose a real number uniformly at random from $[2/3, 1]$. Think about how you would use inversion sampler to simulate a sample x from X, i.e., how you would transform IID samples from Uniform$(0, 1)$ RV to one sample from X.

15. For a given ordered pair of parameters $(k, \lambda) \in (0, \infty)^2$, the RV X is said to be Weibull(k, λ) distributed if its DF is:

$$F(x; k, \lambda) = 1 - \exp\left(-\left(\frac{x}{\lambda}\right)^k\right), \qquad x \in [0, \infty) \ .$$

We want to be able to use the inversion sampler to simulate IID samples from $X \sim$ Weibull(k, λ).

(i) Find $F^{[-1]}(x; k, \lambda)$

(ii) Use this to devise an algorithm that can transform IID samples from Uniform$(0, 1)$ into IID samples from $X \sim$ Weibull(k, λ).

Here are some more probability questions if you feel unsure on this subject

16. If A and B are events, and $A \cap B = \emptyset$ ($P(A \cap B) = 0$), then A and B are:

(A) Independent

(B) Mutually exclusive

17. If A_1 and A_2 are events, and $P(A_1 \cap A_2) = P(A_1)P(A_2)$, then A_1 and A_2 are:

(A) Independent

(B) Mutually exclusive

18. If A and B are events, and $P(A) > 0$, then the definition of conditional probability says that $P(B|A) =$

(A) $\frac{P(B \cap A)}{P(A)}$

(B) $\frac{P(B \cup A)}{P(A)}$

(C) $\frac{P(B \cap A)}{P(B)}$

(D) $\frac{P(B)}{P(A)}$

19. If A_1 and A_2 are events, and $P(A_1 \cap A_2) = P(A_1)P(A_2)$, then

(A) $P(A_2|A_1) = 0$ and $P(A_1|A_2) = 0$

(B) $P(A_1|A_2) = P(A_1)$ and $P(A_2|A_1) = 0$

(C) $P(A_2|A_1) = P(A_2)$ and $P(A_1|A_2) = 0$

(D) $P(A_2|A_1) = P(A_2)$ and $P(A_1|A_2) = P(A_1)$