Markov processes

State space \( \mathbb{R}_0^+ \)

Matrix/transformation/substitution matrix \( M \)

\[
M = \begin{bmatrix}
\alpha & b \\
-\alpha & \lambda
\end{bmatrix}
\]

\( \alpha = \text{prob. of transition} \)

from \( 0 \to 1 \)

\( (\text{branching, e.g., and replication at all levels}) \)

\( \lambda \text{ is a function, the von Neumann \( \lambda \) in the abstract} \)

Tangents at the limit, \( a, b \to 0 \) (as we get closer and closer to the branch, \( b \to 0, a \to 0 \))

\[
L_a := \frac{\partial M}{\partial a} \bigg|_{a=0, b=0}
\]

\[
L_b := \frac{\partial M}{\partial b} \bigg|_{a=0, b=0}
\]

\[
= \begin{bmatrix}
0 & 1 \\
1 & 0
\end{bmatrix}
\]

Target space is a vector space, i.e., \( Q = a L_a + b L_b \)

\[
= \begin{bmatrix}
\alpha - \beta
\end{bmatrix}
\]

\( \beta > 0 \) (also, vector

Jordan space normalizable and arbitrary numbers)

Copy of target space at each point

What happens after branching event, where Markov process is not independent across branches.

What this leads to is:
What about the branch and bound?

What is the true branching?

\[ P \left( [\psi]_{\text{true}} \right) \]

[Diagram with branching nodes and arrows]

Branching event leads to \( p' \to p \).

What is the true operator \( P \) for this node \( p \to p' \)?

What does it mean? We have:

\[ P_d = \{ P_i \text{ if } ... \} \]

\[ P = \left[ \begin{array}{c} \psi_0 \text{ } \psi_1 \end{array} \right] \quad \forall \left[ \begin{array}{c} \psi_0 \text{ } \psi_1 \end{array} \right] \in \mathbb{C}^2 \]

\[ \psi_0 = a |0\rangle + b |1\rangle \]

\[ \psi_1 = a |0\rangle + b |1\rangle \]

This leads to:

\[ p \iff C^2 \otimes C^2 \]

What about "branching operators"?

We need \( \tilde{S} : C^2 \to C^2 \otimes C^2 \)

However, check that \( \tilde{S} (|1\rangle) = |1\rangle \otimes |1\rangle = (|11\rangle) \)

does the right thing, i.e., \( p \iff \tilde{S}(p) \)

Let's do it:

\[ \tilde{S}(p) = \tilde{S}(p_0 |0\rangle + p_1 |1\rangle) \]

\[ = p_0 \tilde{S}(|0\rangle) + p_1 \tilde{S}(|1\rangle) \]

\[ = p_0 (|0\rangle + |1\rangle) + p_1 (|1\rangle + |1\rangle) \]

Conclusion:

\[ \tilde{S}(p) \]

is indeed:

\[ \tilde{S}(p) \]

do the right thing
Before branching: \( \partial_L^m (p) \) \[ \text{Interchanging: } \quad L, J (p) \]

\( \partial_L^m = L J \)

One can check: As operators from \( C^2 \rightarrow C^2 \otimes C^2 \),

\[
\mathcal{E}_m = (L_x \otimes L_y + L_z \otimes L_z + \Delta_x L_x) J
\]

i.e., \( \mathcal{E}_m = L_x \otimes L_y + L_z \otimes L_z + \Delta_x L_x \).

So now let's get out of the "blow up".

Standard description:

\[
M = \begin{bmatrix} \pi_0 & \pi_1 \\ \pi_2 & \pi_3 \end{bmatrix}
\]

\( P = \delta P \cdot \delta M \cdot \delta \pi \cdot \delta \pi \cdot \delta M \cdot \delta \pi \cdot \delta \pi \cdot \delta M \cdot \delta \pi \)
If we take \( n = 1 \), then
\[
\frac{\mathcal{D} \rho}{\partial \rho} = \frac{\partial}{\partial \rho} \left( \frac{\rho}{\sqrt{\alpha}} \right) = \frac{\partial}{\partial \rho} \left( \frac{\rho^2}{\sqrt{\alpha}} \right)
\]

- blah blah

Homomorph: Let \( \xi, \eta, \xi', \eta' \) satisfy some algebraic relations in \( (\xi, \eta) \),

\[
\xi' = -\xi, \quad \eta' = -\eta
\]

\[
\xi \eta' = \left[ \begin{array}{cc} 1 & 1 \\ 0 & 1 \end{array} \right] \left[ \begin{array}{cc} 1 & 1 \\ 0 & 1 \end{array} \right] = \left[ \begin{array}{cc} 2 & 1 \\ 0 & 1 \end{array} \right] : \xi, \eta
\]

\[
\xi \xi' = 1
\]

(And, \( \xi' \eta' \), \( \eta' \xi' \), etc., analogous.)

We have \( d \sigma = \tilde{m} \rho, \tilde{n}, \tilde{s} = d \mu_2 \)

\[
\begin{align*}
\tilde{m} & = \frac{\mathcal{D} \rho}{\partial \rho} \\
\tilde{n} & = \frac{\mathcal{D}_\rho \mathcal{D}_n \rho}{\partial \rho} \\
\tilde{s} & = \frac{\mathcal{D}_\rho \mathcal{D}_\eta \rho}{\partial \rho}
\end{align*}
\]

\[
\begin{align*}
\tilde{m} & = \frac{\mathcal{D} \rho}{\partial \rho} \\
\tilde{n} & = \frac{\mathcal{D}_\rho \mathcal{D}_n \rho}{\partial \rho} \\
\tilde{s} & = \frac{\mathcal{D}_\rho \mathcal{D}_\eta \rho}{\partial \rho}
\end{align*}
\]

Ref.: David Brydine: The energy function (fundamental).

Usually, \( \tilde{s} \) for group based results whereas

Terry's approach holds for all

- blah
This morning:

\[ M = \mathcal{L}_6 \]
\[ Q = \alpha L_x + \beta L_y \]
\[ z = \alpha \begin{bmatrix} -1 & 0 \\ 1 & 0 \end{bmatrix} + \beta \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix} \]

\[ \delta \cdot L_x \{ p^3 \} : \]
\[ \delta \cdot L_x = L_x \cdot \delta = (L_x \otimes L_x + L_x \otimes L_x + L_x \otimes A) \]
\[ \delta L_x : C^2 \to C^2 \otimes C^2 \]
\[ L_x \cdot \delta : C^2 \to C^2 \otimes C^2 \to C^2 \otimes C^2 \]

\[ \delta M_{12} \otimes M_{12} = \delta e \rightarrow \delta (A + \sum_{n=1}^{\infty} \frac{Q_n t^n}{n!}) \]
\[ = (A + \delta (L_x \otimes L_x + L_x \otimes L_x + L_x \otimes A)) \cdot \delta \cdot M_{12} \cdot \delta \]

Here, first one sequence and suddenly two from bifurcation on. Don't matter as sequence are always identical in our case! Each state has two after.

\[ M_{12} : \]

\[
\begin{array}{cccc}
0 & 1 & 0 & 0 \\
0 & 0 & -1 & 0 \\
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 \\
1 & 0 & 0 & 1 \\
1 & 0 & 0 & 1 \\
1 & 1 & 0 & 0 \\
1 & 1 & 0 & 1 \\
1 & 1 & 0 & 1 \\
\end{array}
\]
Recall: \( \hat{M}_n = e^{(\alpha_1, \ldots, \alpha_n) \cdot} \) in previous example

So: Now \( \hat{M}_{13} = e^{(\alpha_{13}, \ldots) \cdot} \)

\[
\hat{M}_{13} = e^{(\alpha_{13}, \ldots) \cdot}
\]

\[
L_{123}^{(123)} = L_{\xi} \otimes L_{\xi} \otimes L_{\xi} + L_{\xi} \otimes L_{\xi} \otimes 1 + L_{\xi} \otimes 1 \otimes L_{\xi}
\]

\[
+ L_{\xi} \otimes L_{\xi} \otimes 1 + L_{\xi} \otimes 1 \otimes L_{\xi} + 1 \otimes 1 \otimes L_{\xi}
\]

\[
\text{perms} = 2^3 - 1
\]

\[
= \sum_{A \subseteq \{1, 2, 3\}} L_{\xi}^{(A)}
\]

\[
A \subseteq \{1, 2, 3\}, A \neq \emptyset
\]

\[
\text{(Remember: } L_{\xi} = L_{\xi} \otimes L_{\xi} + L_{\xi} \otimes 1 + 1 \otimes L_{\xi} \text{)}
\]

\[
= L_{\xi}^{(123)} + L_{\xi}^{(13)} + L_{\xi}^{(23)}
\]

\[
\delta^2 \pi \in C^2 \otimes C^2 \otimes C^2,
\]

\[
[\delta^2 \pi]_{ijk} = \begin{cases} 0 & \text{if } i < j < k \\ \delta_{ik} & \text{if } i = j = k \\ \delta_{ij} & \text{if } i = k, j \neq k = 0 \\ 0 & \text{else} \end{cases}
\]

\[
\delta^2 \pi \rightarrow M_{13} \cdot \delta^2 \pi
\]

\[
\text{Epoch 1: } M_1 \otimes \hat{M}_{23} \cdot \hat{M}_{13} \cdot \delta^2 \pi
\]

\[
\text{Epoch 2: } M_1 \otimes M_2 \otimes \hat{M}_3 \cdot M_1 \otimes \hat{M}_{23} \cdot \hat{M}_{13} \cdot \delta^2 \pi
\]

Now assume in Epoch 4
take 1 & 2 have the same process

\[
\text{Epoch 5: } M_1 \otimes M_2 \otimes M_3 \cdot M_1 \otimes M_2 \otimes M_3 \cdot \delta^2 \pi
\]

No ink here, trebled
& plotted on other states in L_\xi excide
123 alike: Protons on A & C, others
will make identical sequence.
In the limit, A & B will be identical again.

Because nothing sends them out of the identical states in the Markov diagram!