```
> plane_wave:=u(x,t)=A*exp(I*(k*x-omega*t));
```

$$plane\_wave := u(x,t) = A\,e^{I(kx-\omega t)}$$

```
> eval(diff(u(x,t),t)=D*diff(u(x,t),x$2),plane_wave);
```

$$-I A \omega\, e^{I(kx-\omega t)} = -D A k^2 e^{I(kx-\omega t)}$$

```
> omega=solve(%,omega);
```

$$\omega = -I D k^2$$

is the dispersion relation and the solution is

```
> eval(plane_wave,%);
```

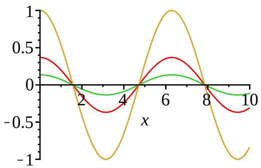
$$u(x,t) = A\,e^{I(kx + I D k^2 t)}$$

```
> expand(%);
```

$$u(x,t) = \frac{A\,e^{Ikx}}{e^{D k^2 t}}$$

thus the amplitude decays exponentially but the wave speed is 0.

```
> plot({seq(eval(Re(rhs(%)),{D=1,A=1,k=1}),t={0,1,2})},x=0..10);
```



Note Re( ) is the real part.

```
> eval(diff(u(x,t),t$2)-c^2*diff(u(x,t),x$2)=0,plane_wave);
```

$$-A\omega^2 e^{I(kx-\omega t)} + c^2 A k^2 e^{I(kx-\omega t)} = 0$$

```
> omega=solve(%,omega);
```

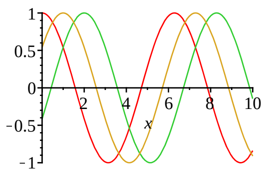
$$\omega = (c\,k,\ -c\,k)$$

```
> eval(plane_wave,omega=c*k);
```

$$u(x,t) = A\,e^{I(kx - ckt)}$$

Thus the amplitude remains constant. The wave travels with constant speed c.

```
> plot({seq(eval(Re(rhs(%)),{c=1,A=1,k=1}),t={0,1,2})},x=0..10);
```




```
> eval(diff(u(x,t),t)+diff(u(x,t),x$3)=0,plane_wave);
```

$$-I A \omega\, e^{I(kx-\omega t)} - I A k^3 e^{I(kx-\omega t)} = 0$$

```
> omega=solve(%,omega);
```

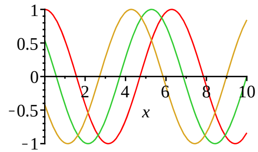
$$\omega = -k^3$$

```
> eval(plane_wave,%);
```

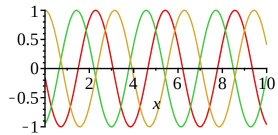
$$u(x,t) = A\,e^{I(kx + k^3 t)}$$

In this case the speed of the wave depends on wave number (that is inversely of wave length). Amplitude remains constant.

```
> plot({seq(eval(Re(rhs(%)),{A=1,k=1}),t={0,1,2})},x=0..10);
```




```
> plot({seq(eval(Re(rhs(%%)),{A=1,k=2}),t={0,1,2})},x=0..10);
  # in this case the wave speed is 8X that of the above
```




```
> eval(diff(u(x,t),t)=I*diff(u(x,t),x$2),plane_wave);
```

$$-I A \omega\, e^{I(kx-\omega t)} = -I A k^2 e^{I(kx-\omega t)}$$

```
> omega=solve(%,omega);
```

$$\omega = k^2$$

```
> eval(plane_wave,%);
```

$$u(x,t) = A\,e^{I(kx - k^2 t)}$$

```
> plot({seq(eval(Re(rhs(%)),{A=1,k=1}),t={0,1,2})},x=0..10);
```