SON2014 (University of Canterbury, NZ)

Abstracts and titles

Accurate control of hyperbolic trajectories in any dimension

Sanjeeva Balasuriya (Adelaide)

The generalisation of attracting, repelling and saddle-like fixed points to nonautonomous (unsteady) flows are called hyperbolic trajectories, which strongly influence transport through their attached stable and unstable manifolds. The fact that all these entities move around in time leads to a richness of behaviours, and challenges in identifying and manipulating them. Here, we address the issue of forcing a hyperbolic trajectory to follow a user-prescribed time-variation. We develop the theory to choose the control velocity in order to achieve accuracy to as high an order as desired. The efficacy of the method is demonstrated by controlling the origin of the Lorenz system, and mixing-enhancing trajectories in a Stokes model for droplets.

Coauthor: Kathrin Padberg-Gehle

Equation-free approach for coarse-graining the dynamics of bursting neurons

Alona Ben-Tal (Massey University, Albany)

A system of bursting neurons often exhibits complex dynamics which consist of multiple states and chaotic behaviour. However, in certain applications, the exact details of this dynamics may not be important and one would like to use a simplified model that captures the dynamics roughly. We have developed a numerical method that maps between the variables of a bursting neural network (for which the equations are known) and the variables of a simplified model (for which the equations are unknown). By moving backward and forward between the variables of the detailed neural network and the variables of the simplified system using restriction and lifting operators, and simulating the detailed neural network for short periods of time, we can calculate the steady states of the simplified model and their stabilities and create bifurcation diagrams for the simplified model even though the equations of the simplified model are unknown. Using this approach we created a coarse bifurcation diagram for a single neuron, consistent with a 2D dynamical system and show how it is related to the bifurcation diagram of the detailed system. We also show how our method could be applied to a network of neurons.

Co-author: Ioannis Kevrekidis

Variational approach to approximation of invariant densities

Chris Bose (Victoria, Canada)

Some time ago we introduced the idea of solving the Perron-Frobenius fixed-point equation through a sequence of moment constrained, convex optimization problems. Computationally, the method is quite comparable (complexity-wise) to Ulam's method, however there is a powerful convergence theory for these problems that guarantees the method produces a rigorous approximation scheme in a wide variety of dynamical settings.

I will begin with a brief review this approach, considering two convex objectives that have proved to be useful: 'energy' and 'entropy'.

Recently we looked at an alternate variational formulation, treating the Perron-Frobenius equation as a regularized least squares problem. On the face of it, this appears to lead to a completely different class of problems, however we can show that the solutions of our finite moment-constrained problems above actually arise as the $\alpha = 0$ limit of the least squares solutions (where α is the regularization parameter, and for suitably chosen regularization terms). Perhaps even more interesting, Ulam's method can also be derived this way.

In this preliminary report I will describe our approach, how it connects to the previously studied moment problems and outline some ideas around convergence of the approximate solutions obtained this way.

Coauthor: Rua Murray.

The Lorenz system near the loss of the foliation condition.

Jennifer L Creaser (Auckland)

The well-known Lorenz system is classically studied via its reduction to the one-dimensional Lorenz map. To capture the full behaviour of the dynamics of the system, this reduction requires the existence of a stable foliation. We study a parameter regime where this so-called foliation condition fails for the first time and, subsequently, the Lorenz map no longer accurately represents the dynamics. To this end, we study how the three-dimensional phase space is organised by the global invariant manifolds of saddle equilibria and saddle periodic orbits.

Specifically, we explain the development of hooks in the Poincaré return map that marks the loss of the foliation condition. We make extensive use of the continuation of orbit segments formulated by two-point boundary value problems to calculate the intersection curves of the two-dimensional unstable manifold $W^u(\Gamma)$ of a periodic orbit Γ with the Poincaré section. We formulate, identify and calculate when hooks form in the Poincaré map as a point of tangency of $W^u(\Gamma)$ with the stable foliation.

Coauthors: Bernd Krauskopf and Hinke M Osinga

Homogenisation of homo- and hetero-cellular coupling

Tim David (Bluefern Supercomputing Unit, Canterbury)

Biological structures are heterogeneous on many scales such as nano scale, micro scale and macro scale. An attempt to incorporate these structures poses difficulties to the mathematical modeller because of the computational size of the model. Multiscale analysis and the theory of homogenization provide the mathematical modeller with a way forward, avoiding the need to perform large multiscale calculations.

Our group has developed a system of reaction-diffusion equations to study the spatial variation of homogenic (cells of same type) and heterogenic (cells of different type) coupling between Endothelial cells (ECs) and Smooth muscle cells (SMCs), in terms of fast and slow variables. Moreover, by employing asymptotic analysis and the theory of homogenization, one-dimensional homogenized partial differential equations (HPDE) for the homogenic coupling of membrane potential and calcium dynamics in SMCs were derived. Furthermore, an effective diffusion coefficient was also derived analytically. Numerical simulations were undertaken to ensure oscillations and wave propagation incorporating the circumferential coupling and axial diffusion.

Keywords: micro and macro scales, homogeneity, heterogeneity, bifurcation, asymptotic analysis, homogenisation.

Computing coherent sets using the Fokker-Planck equation

Andreas Denner (TU Munich)

The computation of sets in the phase space of a time dependent dynamical system which are separated by transport barriers, so called coherent sets, is of interest in, e.g., atmospheric flows and plasma physics. In this talk we present an alternative to Froyland's method for computing coherent sets which is based on a continuous diffusion in time, i.e. the Fokker-Planck equation. We approximate the resulting transfer operator family using spectral collocation. The talk is completed with some numerical examples.

Finite-time coherent sets – with and without diffusion

Gary Froyland (UNSW)

Because fluid flow is invertible, existing approaches to the identification of those regions in the flow which remain coherent over finite time durations have required a small amount of diffusion. These approaches work very well because interactions between coherent sets and the rest of the phase space are strongest near the boundaries of the coherent sets. The use of diffusion is also very convenient computationally, as it arises naturally through Ulam-type (set-oriented) discretisations. However, for purely deterministic fluid flow, the diffusion makes comparisons with geometric objects difficult. In this presentation we describe a "new" approach for identifying coherent sets for deterministic flows, which has no diffusion requirement. This allows us to make reasonable strong geometric statements about the identified coherent sets, and their connection with the eigenvalues and eigenfunctions of a new linear operator based on the transfer operator. I will show that this "new" approach, while previously unknown (at least to the speaker), is not really new after all and fits naturally alongside the diffusive transfer operator constructions.

Global invariant manifolds near a homoclinic flip bifurcation

Andrus Giraldo (Auckland)

Homoclinic bifurcations, such as the well-known Shilnikov bifurcation, may give rise to complicated dynamics including oscillations and chaos. Our work is on the homoclinic flip bifurcation, which can be found when two parameters are varied. Its characterising feature is that the associated stable (or unstable) manifolds change from being orientable to non-orientable. The homoclinic flip bifurcation gives rise to different bifurcation curves. We are interested in the organisation of the global manifolds in a three-dimensional model vector field as these curves are crossed.

This talk will focus on explaining the unfolding of the homoclinic flip bifurcation in a particular case, with a bifurcating saddle periodic orbit, for which the configuration of invariant manifolds has not been studied before. In our study the invariant manifolds are computed via the continuation of solutions of a two-point boundary value problem.

Non-autonomous dynamical systems and multiplicative ergodic theorems

Cecilia González-Tokman (UNSW)

Non-autonomous (or random) dynamical systems yield very flexible models for the study of time-dependent systems, with driving mechanisms allowed to range from deterministic forcing to stationary noise. Multiplicative ergodic theorems (METs) encompass fundamental information for the study of transport phenomena in such systems, including Lyapunov exponents, invariant measures and coherent structures.

In this talk we will motivate and discuss recent developments on METs. We will then address related stability questions, which arise naturally in the context of non-autonomous systems from the use of numerical approximation schemes, as well as from the presence of modelling errors and noise. (This talk is based on joint work with Gary Froyland and Anthony Quas.)

Dynamics of micro-mechanical arrays

Stefanie Gutschmidt (Canterbury)

An arrayed architecture of micro beams is a promising approach to increase the speed of scanning proximity microscopes and related technologies. These arrays require sensors and actuators to be integrated into each beam, which results additional physical fields or different properties for the individual beams. Thus, well established modelling approaches for single beams need to be reassessed in view of its applicability for arrays. This work considers an array of micro-cantilevers, where each oscillator is equipped with a thermal actuator and a piezo-resistive displacement measurement. To draw conclusions on the movement of the beam's free end based on a measurement of the mechanical stress at the fixed end, a comprehensive reduced order model is derived. The model incorporates the beam's composite structure and varying geometry along its length and is coupled to a thermal model for actuation as well as thermo-elastic damping. Subsequently, the influence of the beam's geometry on its eigenfunctions and -frequencies is analysed, including force interaction resulting from an operation in close proximity to a surface. Occurring internal resonances are linked to geometrical and interaction properties. A dynamical analysis focuses on a prediction of the motion of the beams' free end based on the measured mechanical stress as well as the beams' behaviour for varying excitations.

Bifurcations of invariant sets in a map model of wild chaos

Stefanie Hittmeyer (Auckland)

We study a two-dimensional noninvertible map that has been introduced by Bamon, Kiwi and Rivera in 2006 as a model of wild Lorenz-like chaos. The map acts on the plane by opening up the critical point to a disk and wrapping the plane twice around it; points inside the disk have no preimage. The bounding critical circle and its images, together with the critical point and its preimages, form the so-called critical set. This set interacts with the stable and unstable sets of a saddle fixed point and other saddle invariant sets. Advanced numerical techniques enable us to study how these invariant sets change as the parameters are varied towards the wild chaotic regime. We find a consistent sequence of four types of bifurcations, which we present as a first attempt towards explaining the geometric nature of wild chaos. In a different parameter regime, the map acts as a perturbation of the complex quadratic family and admits (a generalised notion of) the Julia set as an additional invariant set. When parameters are varied, this set interacts with the other invariant sets, leading to the (dis)appearance of saddle points and chaotic attractors and to dramatic changes in the topology of the Julia set. In particular, we find generalised Julia sets in the form of Cantor bouquets, Cantor tangles and Cantor cheeses. Using two-parameter bifurcation diagrams, we show that the same sequences of bifurcations occur along different paths towards the wild chaotic regime and we use this information to obtain an indication on the size of the parameter region where wild chaos is conjectured to exist. In addition, we reveal a self-similar bifurcation structure near the period-doubling route to chaos in the complex quadratic family.

Coherence in augmented space

Péter Koltai (FU Berlin)

The decomposition of the state space of a dynamical system into almost invariant (or metastable) sets is important for understanding its essential macroscopic behavior, hence for complexity reduction. The concept is quite well understood for autonomous dynamical systems, and recently steps have been made to generalize it for non-autonomous systems. The associated objects are called coherent sets. Apart from the common idea underlying their definition, it is natural to ask which connections are there between (the autonomous) almost invariant sets and (the non-autonomous) coherent sets. The motivation is twofold: transferring the theory developed for the autonomous case, and devising new, more efficient numerical methods for the computation of coherent sets. In this talk we present first steps towards answering the above question, considering the system augmented by the time component. Our results will be restricted to systems with time-periodic forcing.

A boundary value approach to computing slow manifolds and canard orbits

Bernd Krauskopf (Auckland)

We present a general technique for the computation of two-dimensional slow manifolds in systems with one fast and two slow variables. It is based on the continuation of a one-parameter family of orbit segments, given as solutions of a suitably-defined boundary value problem. In this way, we are able to deal effectively with the numerical challenges of strong attraction to and strong repulsion from the slow manifolds. Visualisation of the computed surfaces gives unprecedented insight into the geometry of the system. In particular, our technique allows us to find and then continue canard solutions as the intersection curves of attracting and repelling slow manifolds. The method is illustrated with the self-coupled FitzHugh–Nagumo system, and a reduced Hodgkin–Huxley model. We concentrate on mixed-mode periodic orbits that arise geometrically near a folded-node singularity. We show how the subsequent continuation of canard orbits allows us to find and investigate new types of dynamics, such as the interaction between canard orbits and a saddle periodic orbit that is generated in a singular Hopf bifurcation.

A Lin's method approach for detecting canard orbits

José Mujica (Auckland)

Canard orbits are relevant objects in the study of multiple-time scale systems since they give information about how slow manifolds spiral around fold curves on the critical manifold. These orbits arise from the intersection between the attracting and repelling slow manifolds. By implementing Lin's method in a boundary value problem setup we are able to detect canard orbits in a systematic way. This is demonstrated with the examples of the normal form for a folded node and the Koper model. We also use this approach for finding intersections between a slow manifold and a global invariant manifold.

Coauthors: Bernd Krauskopf and Hinke M Osinga

Computing global invariant manifolds: techniques and applications

Hinke M Osinga (Auckland)

Global invariant manifolds play an important role in organising the behaviour of a dynamical system. Together with equilibria and periodic orbits, they form the so-called skeleton of the dynamics and offer geometric insight into how observed behaviour arises. In most cases, it is impossible to find invariant manifolds explicitly and numerical methods must be used to find accurate approximations. Developing such computational techniques is a challenge on its own and, to this date, the focus has primarily been on computing two-dimensional manifolds. Nevertheless, these computational efforts offer new insights that go far beyond a confirmation of the known theory. Furthermore, global invariant manifolds in dynamical systems theory not only explain asymptotic behaviour, but more recent developments show that they are equally useful for explaining short-term transient dynamics. We present an overview of these more recent developments, in terms of novel computational methods, as well as applications that have stimulated recent advances in the field and highlighted the need for new mathematical theory.

Coherence in nonautonomous dynamics

Kathrin Padberg-Gehle (TU Dresden)

Transport properties of nonautonomous dynamical systems over a finite-time interval can be described within a probabilistic framework. Of particular interest are coherent sets. These are time-dependent macroscopic structures that hardly mix with the rest of phase space over the considered time span. Such behavior can be observed in many real-world phenomena, including the polar vortex, gyres and eddies in the ocean as well as thermal plumes in convection. Coherent sets can be efficiently detected and approximated by a transfer operator based approach within a set-oriented numerical framework. Here we give an introduction to the theory and numerics of the coherent sets constructions and demonstrate their properties in a number of example systems. We also discuss aspects of controlling the transport and mixing properties of nonautonomous flows.

Multiobjective optimal control methods for the development of an intelligent cruise control

Sebastian Peitz (Paderborn)

During the last years, alternative drive technologies, for example electrically powered vehicles (EV), have gained more and more attention, mainly caused by an increasing awareness of the impact of CO_2 emissions on climate change and by the limitation of fossil fuels. However, these technologies currently come with new challenges due to limited lithium ion battery storage density and high battery costs which lead to a considerably reduced range in comparison to conventional internal combustion engine powered vehicles. For this reason, it is desirable to increase the vehicle range without enlarging the battery. When the route and the road slope are known in advance, it is possible to vary the vehicles velocity within certain limits in order to reduce the overall drivetrain energy consumption. This may either result in an increased range or, alternatively, in larger energy reserves for comfort functions such as air conditioning.

In this presentation, we formulate the challenge of range extension as a multiobjective optimal control problem. We then apply different numerical methods to calculate the so-called Pareto set of optimal compromises for the drivetrain power profile with respect to the two concurrent objectives: battery state of charge and mean velocity. In order to numerically solve the optimal control problem by means of a direct method, a time discretization of the drivetrain power profile is necessary. In combination with a vehicle dynamics simulation model, the optimal control problem is transformed into a high dimensional nonlinear optimization problem. For the approximation of the Pareto set, two different optimization algorithms implemented in the software package GAIO are used. The first one yields a global optimal solution by applying a set-oriented subdivision technique to parameter space. By construction, this technique is limited to coarse discretizations of the drivetrain power profile. In contrast, the second technique, which is based on an image space continuation method, is more suitable when the number of parameters is large while the number of objectives is less than five. We compare the solutions of the two algorithms and study the influence of different discretizations on the quality of the solutions.

A MATLAB/Simulink model is used to describe the dynamics of an EV. It is based on a drivetrain efficiency map and considers vehicle properties such as rolling friction and air drag, as well as environmental conditions like slope and ambient temperature. The vehicle model takes into account the traction battery too, enabling an exact prediction of the batterys response to power requests of drivetrain and auxiliary loads, including state of charge.

Coauthors: Julian Eckstein, Patrick Friedel, Ulrich Köhler, Sebastian Tiemeyer (HELLA) and Michael Dellnitz, Kathrin Flaßkamp, Christian Horenkamp, Sina Ober-Blöbaum (Paderborn)

The topological complexity of orbit data

Jean-Luc Thiffeault (University of Wisconsin - Madison)

Experiments and numerical simulations provide plenty of data describing orbits of systems, dynamical or otherwise. This data is sparse: usually we have access to only a small set of orbits, and set-oriented techniques are unfortunately not ideal. But what can we learn about the underlying motion from these orbits? Here we propose to view such orbits as entanglements, akin to an entangled braid of hair. Tools from computational topology can then be brought to bear on the data. As a baseline, we look at modeling the entanglement of purely random orbits such as Brownian motion. This is joint work with Marko Budisic and Huanyu Wen.

Schedule: SON2014 (University of Canterbury, NZ)

Monday, 1 September

10.00-11.00	Arrival, registration and morning tea		Erskine 447
11.00-12.00	OSINGA		Erskine 446
12.00-1.30	Lunch		Erskine 447
1.30-2.15	BOSE		Erskine 446
2.15-3.00	KRAUSKOPF		Erskine 446
3.00-3.30	Afternoon tea		Erskine 447
3.30-4.00	CREASER		Erskine 446
4.00-5.00	Breakout / Free time		Erskine 505
5.00-6.30	Welcome Reception		UC Staff Club

Tuesday, 2 September

9.30-10.30	THIFFEAULT		Erskine 446
10.30-11.00	Morning tea		Erksine 447
11.00-11.30	GIRALDO		Erskine 446
11.30-12.00	MUJICA		Erskine 446
12.00-1.30	Lunch		Erskine 447
1.30-2.30	PADBERG-GEHLE		Erskine 446
2.30-3.15	GUTSCHMIDT		Erskine 446
3.15	Afternoon tea	(then breakout)	Erskine 447/505

Wednesday, 3 September

9.30-10.30	DAVID		Erskine 446
10.30-11.00	Morning tea		Erksine 447
11.00-11.30	PEITZ		Erskine 446
11.30-12.00	DENNER		Erskine 446
12.00 -	Breakout / free afternoon	(no lunch)	Erskine 505 avail

Thursday, 4 September

9.30-10.30	GONZALEZ-TOKMAN		Erskine 446
10.30-11.00	Morning tea		Erksine 447
11.00-11.45	HITTMEYER		Erskine 446
12.00-1.30	Lunch		Erskine 447
1.30-2.15	BALISURIYA		Erskine 446
2.15-3.00	BEN-TAL		Erskine 446
3.00	Afternoon tea	(then breakout)	Erskine 447/505

Friday, 5 September

9.30-10.30	FROYLAND		Erskine 446
10.30-11.00	Morning tea		Erskine 447
11.00-11.45	KOLTAI		Erskine 446
11.45-12.00	Closing		Erskine 446
12.00	Lunch	(Erskine 505 avail)	Erskine 447