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document center**Lang's conjectures, fibered powers, and uniformity**

Introduction Let X be a variety of general type defined over a number field K. It was conjectured by S. Lang that the set of rational points $X(K)$ is not Zariski dense in X. In the paper chm of L. Caporaso, J. Harris and B. Mazur it is shown that the above conjecture of Lang implies the existence of a uniform bound on the number of K-rational points of all curves of fixed genus g over K.

The paper has immediately created a chasm among arithmetic geometers. This chasm, which sometimes runs right in the middle of the personalities involved, divides the loyal believers of Lang's conjecture, who marvel at this powerful implication, and the disbelievers, who try to use this implication to derive counterexamples to the conjecture.

In this paper we will attempt to deepen this chasm on both sides: first, using the techniques of chm and continuing abr, we prove more implications, some of which are very strong, of various conjectures of Lang. Along the way we will often use the Fibered Power Conjecture, also known as Conjecture H (see chm, §6) about higher dimensional varieties, which is regarded as very plausible among experts of higher dimensional algebraic geometry.

Second, we will show by way of counterexamples that two natural candidates for analogous statements in positive characteristic, are false.

Before we state any results, we need to specify various conjectures which we will apply.

A few conjectures of Lang Let X be a variety of general type over a field K **of characteristic 0**. In view of Faltings's proof of Mordell's conjecture, Lang has stated the following conjectures:

conj enumerate

(Weak Lang conjecture) If K is finitely generated over Q then the set of rational points $X(K)$ is not Zariski dense in X.

(Weak Lang conjecture for function fields) If $k \subset K$ is a finitely generated regular extension in characteristic 0, and if $X(K)$ is Zariski dense in X, then X is birational to a variety X_0 defined over k and the "non-constant points" $X(K) \setminus X_0(k)$ are not Zariski dense in X.

(Geometric Lang conjecture) Assuming only $Char(K) = 0$, there is a proper Zariski closed subset $Z(X) \subset X$, called in chm the Langian exceptional set, which is the union of all positive dimensional subvarieties which are not of general type.

(Strong Lang conjecture) If K is finitely generated over Q then there is a Zariski closed subset $Z \subset X$ such that for any finitely generated field $L \supset K$ we have that $X(L) \setminus Z(L)$ is finite.